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The quantum Zeno effect and quantum feedback in cavity QED

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Abstract
We explore experimentally the fundamental projective properties of a quantum measurement and their application in the control of a system's evolution. We perform quantum non-demolition (QND) photon counting on a microwave field trapped in a very-high-Q superconducting cavity, employing circular Rydberg atoms as non-absorbing probes of light. By repeated measurement of the cavity field we demonstrated the freeze of its initially coherent evolution, illustrating the back action of the photon number measurement on the field's phase. On the contrary, by utilizing a weak QND measurement in combination with the control injection of coherent pulses, we plan to force the field to deterministically evolve towards any target photon-number state. This quantum feedback procedure will enable us to prepare and protect photon-number states against decoherence.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction

Any measurement performed on a quantum system produces an unavoidable back action on its state. Being well controlled, a quantum measurement can not only project the system's state onto an arbitrary eigenstate of the corresponding measurement operator, but also be used to influence the system's evolution. We investigate these effects experimentally by using a quantum non-demolition (QND) measurement of the number of photons in a field, trapped in a high-quality microwave cavity [1, 2]. The cavity field is probed by a stream of non-absorbing circular Rydberg atoms acting as quantum probes of light [3, 4]. Combining the QND measurement with coherent microwave injection into the cavity, we first demonstrate the quantum Zeno effect ([5]; [6] and references therein; [7]). By repeatedly injecting tiny coherent pulses into the cavity mode, we observe the coherent growth of the field amplitude. This evolution is dramatically inhibited by a repeated measurement of the photon number performed between injections. This freezing of the coherent field evolution illustrates the back action of the photon-number determination onto the field phase, which becomes completely blurred after one measurement, thus making impossible the coherent addition of a subsequent injection pulse.

The QND photon counting performed on an initial coherent field prepares a random photon-number (Fock) state of light. We propose an active quantum feedback scheme to deterministically generate any desired Fock states and to protect them against decay [8, 9]. In the feedback loop, detection of each individual atom provides us with information on the current state of the field. We then steer it towards a target state by injecting into the cavity a coherent pulse adjusted to increase the population of the

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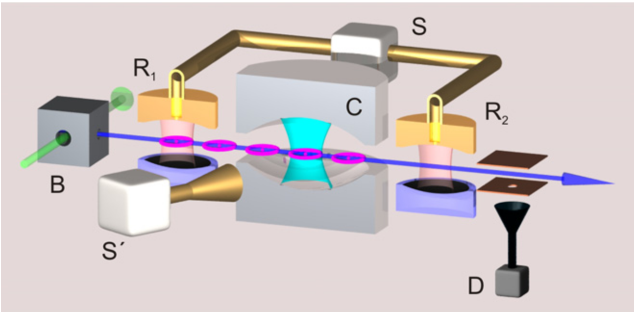

Figure 1. Sketch of the QND photon counting setup. Single Rydberg atoms are prepared in box B. After interacting one by one with the mode of a high-Q superconducting cavity C coupled to microwave source S', the atoms are detecting in a field-ionization detector D. Before and after flying through C, the atoms are exposed to $\pi/2$ pulses in low-Q Ramsey cavities R_1 and R_2, fed by the classical source S.

desired photon number n and to guarantee the convergence of the field towards state $|n\rangle$ after several tens of feedback cycles. Keeping the procedure active after convergence will maintain the field in this Fock state, restoring it after each decoherence-induced quantum jump. The efficiency and reliability of the closed-loop state stabilization are illustrated by the results of quantum Monte-Carlo simulations [10].

2. QND counting of photons

Our non-destructive measurement of the number of photons in the cavity field is based on the dispersive atom–field interaction, preventing the energy exchange between the two systems and, thus, any change in the photon number [11]. In this regime, the atom–field detuning δ is larger than the coupling strength between the systems, i.e. the Rabi frequency Ω_0. While the atom flies through the cavity mode, the interaction leads to a shift of its two states, $|g\rangle$ and $|e\rangle$, in the opposite direction. This light shift, being proportional to the total number n of the field photons, manifests itself as an accumulated phase of the atomic superposition $(|g\rangle + |e\rangle)/\sqrt{2}$. In the frame, rotating with the unperturbed atomic frequency, the Bloch vector presenting the atomic spin thus becomes dephased in the equatorial plane of the Bloch sphere by an angle ϕ, proportional to n.

Our microwave cavity QED setup that allows us to measure ϕ is schematically presented in figure 1. High-Q cavity C, made of two concave superconducting mirrors of high surface quality, has a resonance frequency of 51 GHz and a cavity damping time of $T_C = 130$ ms [12]. A pulsed stream of circular Rydberg atoms in state $|g\rangle$ is excited from ground-state velocity-selected rubidium atoms in the circularization box B. The atom detector D, based on detection of a single electron after ionizing the atom from one of the two quantum states, is obviously not sensitive to the superposition phase ϕ. We therefore utilize the Ramsey interferometer to measure ϕ, which consists of two Ramsey zones (i.e. two low-Q cavities R_1 and R_2 coupled to a coherent source S) placed before and after the dephasing region (i.e. a high-Q cavity C). After being exposed to a $\pi/2$-pulse in R_1, the atom is prepared in the state superposition $(|g\rangle + |e\rangle)/\sqrt{2}$. Before the state detection in D, we apply a second $\pi/2$-pulse in R_2. This scheme allows us to effectively measure the atomic spin along any direction ϕ_R in the equatorial plane defined by the phase ϕ_R of the second $\pi/2$-pulse. So, by continuously changing the Ramsey phase, we observe sinusoidal oscillations of two-state populations, known as Ramsey fringes [11].

Single-atom measurement does not allow us to completely detect the spin direction and, thus, the photon number n present in the field. Therefore, we use a stream of many atoms (typically 50–100) and detect them individually along different directions. The information, collected in this way, completely determines the atomic spin direction and thus results in n. Although the measurement outcome n is random with the probability given by the photon-number distribution of the initial field state, the outcome of the next measurement performed on the same field realization will surely give us the same result n. This property uniquely illustrates the QND nature of our measurement preserving the photon number [1, 2].

3. Freezing coherent field growth by the quantum Zenon effect

3.1. Expected field evolution

As the free evolution of our system we have chosen a coherent growth of the cavity field due to repeated injection into its mode of a series of small resonant microwave pulses with equal complex amplitude $\lambda \ll 1$. After N injections the field's amplitude of the initially empty cavity is thus $\alpha = \lambda N$. The average photon number runs away quadratically with N as $\langle n\rangle = |\alpha|^2 = |\lambda|^2 N^2$. Now, we consider the same pulsed coupling with the source alternated with QND photon counting. Since after the first injection the probability for the field to contain 1 photon is only $|\lambda|^2 \ll 1$, the result of photon measurement will project the field into an initial vacuum state $|0\rangle$ with a high probability of $p_0 \approx 1 - |\lambda|^2$. After N iterations of the injection/measurement sequence, the field is left empty with the probability of $p_{0,N} \approx [1 - |\lambda|^2]^N \approx 1 - |\lambda|^2 N$. Thus, the average photon number in this case grows only linearly with N as $\langle n\rangle = |\lambda|^2 N$ and is much smaller than in the case of the free evolution. If we keep constant the overall evolution time and increase the number of injection pulses N by reducing the pulse duration (and, consequently, λ), the ratio between the two photon numbers can be made arbitrarily large. In the limit of infinitely many measurements, the field thus always remains in $|0\rangle$. This frozen evolution of the field is the striking manifestation of the quantum Zeno effect.

Remarkably, the expected result can also be explained without directly considering the projection of the field state after each measurement. Being a conjugate variable to the photon number, the field phase completely diffuses after each full measurement of n. Thus, instead of being added coherently with the same phase (resulting in a linear growth of amplitude), the injection pulses add randomly. The field amplitude undergoes then a two-dimensional random walk in the phase space and thus grows only as a square root of the number of step injections, leading to a linear increase of the average photon number $\langle n\rangle$.

3.2. Experimental results

The experimental setup used to realize a quantum Zeno effect is the same as shown in figure 1 and the measurement results are presented in more detail in [7]. The cavity field is initially prepared in the vacuum state $|0\rangle$ by sending several tens of atoms in state $|g\rangle$, which resonantly absorb and then take away all cavity photons. Identical microwave injection pulses have a duration of $50\,\mu$s and are separated by 5.04 ms. The average photon number per pulse is $\langle n_1\rangle = |\lambda|^2 = 0.002\,23 \pm 0.000\,12$. The important requirement for the coherent accumulation of successive pulses is phase stability. The lifetime-limited linewidth of C is 1.2 Hz full-width at half-maximum. Since the cavity frequency must be stabilized at this level, the 2.7 cm separation between the cavity mirrors must be stabilized to a 0.5 pm precision. For this purpose, we have first isolated the cryostat containing the experimental setup from external vibrations. Next, the temperature of the mirrors is stabilized to $\pm 10^{-4}$ K and the pressure of the liquid helium bath to ± 0.1 mbar. Finally, we filter the control voltage applied to piezo elements that tune the mirror separation and hence the mode frequency. We measure the cavity frequency every 10 min during data acquisition and reset the source frequency. The estimated average cavity–source detuning is then 0.6 ± 0.2 Hz.

In order to measure the free-field evolution, we perform one QND photon measurement after injecting N microwave pulses. The chosen dephasing per photon of $\Phi_0 \approx \pi/4$ is appropriate for unambiguous counting up to seven photons. The atomic spins are detected along the four different axes corresponding to $\phi/\pi = -0.250$, -0.047, $+0.247$ and $+0.547$. On average, 200 atoms are detected in a 72 ms time interval, which is shorter than the cavity lifetime T_c. For each N, we repeat the experimental sequence 500–2000 times. The measurement results are presented as red squares in figure 2(*a*). As expected, the mean photon number initially grows quadratically. However, due to limited cavity lifetime and residual phase drifts, the signal slowly saturates. The solid (red) line is a fit from which we deduce the precise amplitude of the injection pulses, given before.

Now we fix the number of injections to $N = 100$ and repetitively measure $\langle n\rangle$ with QND atomic probes after each injection. Since the photon number is now expected to remain small, we simplify the QND measurement procedure by using a single detection phase of $\phi/\pi = 0.278$, which optimally discriminates between 0, 1 and 2 photons. We detect on average ten atoms between successive injections, which are not enough to unambiguously detect n. Therefore, we collect atoms from five consecutive injection/measurement cycles around the chosen time in order to completely determine the field amplitude. Since the amplitude injected during five cycles is still much smaller than 1, the applied averaging does not prevent us from observing the quantum Zeno effect. We repeat this sequence several hundred times and average the results obtained. Blue circles in figure 2(*a*) are the measured mean photon number in the presence of frequent QND photon counting. As expected, $\langle n\rangle$ remains very low for all times ($\langle n\rangle < 0.2$), experimentally illustrating the frozen coherent evolution by projective quantum measurement. The vertical zoom, shown in figure 2(*b*), reveals the saturation of $\langle n\rangle$ due to cavity decay. The solid curve is a result of Monte-Carlo simulation of the phase diffusion induced by atom detections, in good correspondence with the measured data. The simulation takes into account cavity relaxation, the finite cavity temperature and the limited efficiency of atom detection.

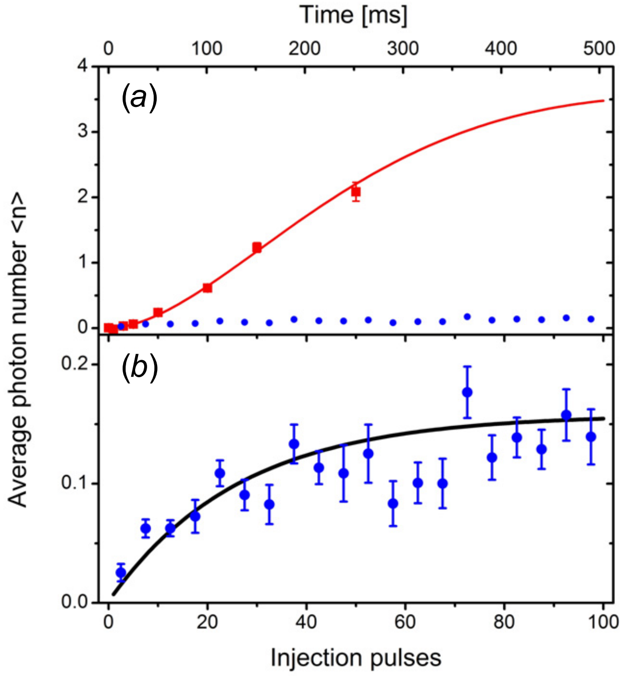

Figure 2. Average photon number in the cavity as a function of the number of injection pulses. Squares (red) and circles (blue) present the field evolution without and with the repetitive QND measurements, respectively. (*b*) A close-up view of panel (*a*). Adapted from [7].

4. Quantum feedback by weak QND measurements

4.1. Proposed quantum feedback scheme

As discussed above, QND measurement randomly and unavoidably projects the field into one of its eigenstates. In our case, the measurement is performed with many individual atomic probes, completely projecting the cavity field onto a random photon number state $|n\rangle$. On the contrary, measurement with only one atom does not completely determine n, although it still modifies our knowledge of the current photon number distribution $P(n)$. Therefore, it can be referred to as a 'weak' measurement of n. Only after repeating a weak measurement many times, the photon-number distribution deduced from every single atom will evolve into a final random Fock state $|n\rangle$.

If we want to avoid randomness in state evolution under QND measurement, the latter should be decomposed into a series of weak measurements, each one providing us with information on the intermediate field state. This information can then be used to design a control action to the field such that the occupation of the target photon number $P(n_{tag})$, is increased before the next weak measurement. As such a control we can use coherent injection of microwave pulses into the cavity mode. By iterating the weak measurement/state-control procedure many times, one should in principle be able to steer the field towards any

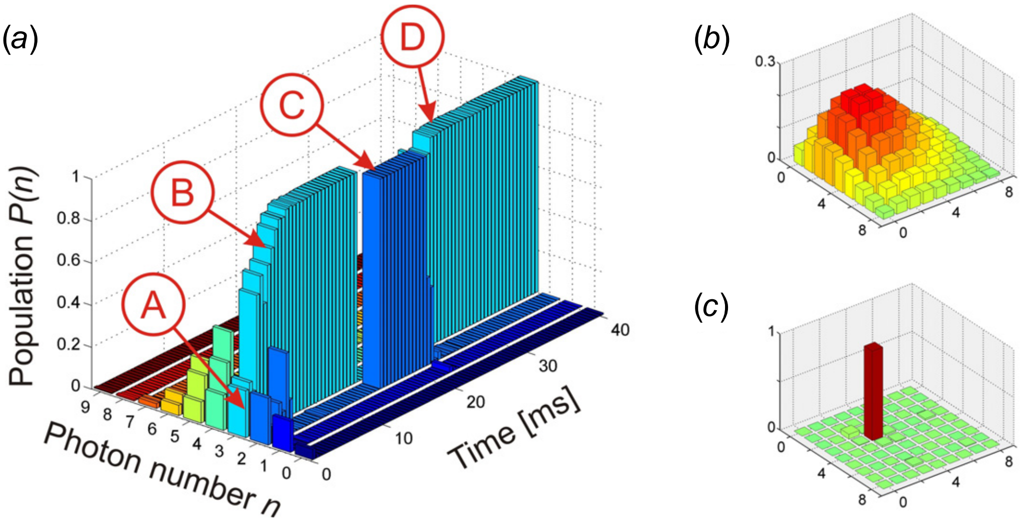

Figure 3. Monte-Carlo simulations of the proposed quantum feedback scheme for realistic experimental parameters with a target three-photon Fock state. (*a*) A single closed-loop quantum trajectory shown as a time evolution of the photon number populations in the cavity state. (*b*, *c*) Density matrices of the initial coherent state and of the final nearly ideal Fock state of the field, respectively.

chosen Fock state. Moreover, by keeping the feedback active, quantum jumps of the field due to sudden loss or generation of thermal photons can be efficiently traced out and corrected, thus partially protecting the field against decoherence. The proposed feedback scheme has all the main components present in any closed-loop system: a sensor (in our case, QND atomic probes in a Ramsey interferometer), a controller (a fast computer calculating required injection) and an actuator (classical microwave source). Using the quantum sensor, we thus have the *quantum* feedback loop.

4.2. Results of Monte-Carlo simulations

The described feedback scheme is well adapted to our microwave cavity QED setup for the preparation of any small-photon-number states $|n_{tag}\rangle$. In order to optimize the feedback protocol and to explore its convergence and stability properties, we have performed Monte-Carlo simulation of the feedback action [9]. The chosen atom–cavity detuning ($\delta/2\pi = 238\,\text{kHz}$) and the interaction time (atomic velocity $v = 200\,\text{m s}^{-1}$) have been chosen to provide single-photon detuning of about $\pi/7$, allowing us to discriminate n for up to 8 photons. The Ramsey phase is set to the mid-fringe position with respect to the target photon number n_{tag} in order to provide the largest sensitivity to it.

We consider all significant features and imperfections of our cavity QED system. Pulsed Rydberg-state excitation from a continuous beam of ground-state atoms prepares atomic samples every 85 μs with a Poisson distribution of atom number. In order to reduce the probability of simultaneous interaction of several atoms with the field, we keep a single atom occupation low (about 0.3 atom per sample). The detection efficiency of about 80% represents an additional source of imperfection. Next, the finite contrast of the Ramsey interferometer results in about 0.1 probability of erroneous state assignation. Finally, in all simulations we must take into account a limited cavity lifetime of 130 ms and its finite temperature of 0.8 K, resulting in the equilibrium thermal photon number of 0.05.

Figure 3(*a*) shows a typical quantum trajectory of the cavity quantum state as the time evolution of photon number probabilities $P(n)$ of the field. We chose to prepare the three-photon state. Initially (**A**) the cavity field is prepared in a coherent state with a mean photon number $\langle n\rangle = n_{tag} = 3$. The corresponding density matrix is shown in figure 3(*b*). Soon after switching on the feedback, the occupation of the three-photon state starts to dominate over other *n*s and quickly reaches unity (**B**). After feedback convergence, the prepared state survives until sudden photon loss due to limited cavity lifetime (**C**). However, by continuing to run feedback loops, a quantum jump is soon detected and efficiently compensated (**D**). As a result, we get a desired photon number state of a high purity, as shown in figure 3(*c*).

More detailed studies show that due to sudden photon jumps, the average fidelity of the cavity state at any time with respect to $|n_{tag}\rangle$ is about 63%. Figure 4(*a*) shows the average over 10^4 quantum trajectories converging to this level. On the other hand, if one does not require the state to be prepared at a fixed time, the fidelity of Fock-state preparation can be arbitrarily high. So, with the success probability of better than 90%, the cavity reaches the target state (with 95% fidelity) at some point within the first 85 ms of feedback operation.

The evolution of the field after a sudden photon lost is presented in figure 4(*b*). After simulating many trajectories, we take 10^4 of them that show a jump after the field has converged to $|n_{tag}\rangle$. Then, we average the trajectories by shifting their time origins to the moment of their jump. Since the jump was from the state $|n_{tag} = 3\rangle$, the shown field starts from $|2\rangle$. After about 5 ms the feedback algorithm starts to react on the state change and then the desired state is recovered after about 10 ms.

5. Conclusion

The microwave cavity QED setup, presented here, provides us with a very powerful tool to explore the basic postulates of quantum mechanics. QND photon counting, which was successfully realized several years ago in our group, has

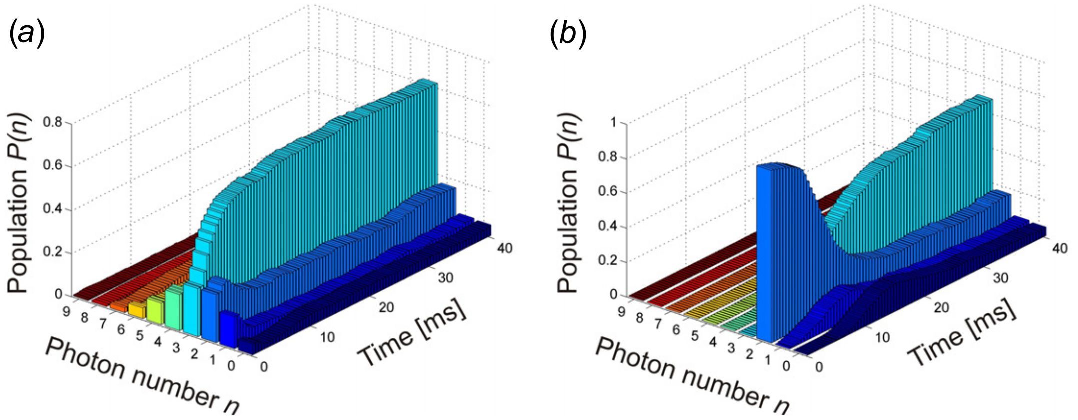

Figure 4. Convergence properties of the quantum feedback. (*a*) Average over 10^4 individual quantum trajectories similar to that of figure 3. It takes about 10 ms for converging to the target state. The reduced state fidelity of 63% is due to limited cavity lifetime, resulting in sudden and unavoidable photon number jumps. (*b*) Average over 10^4 quantum trajectories, each starting from a quantum jump from the desired Fock state $|3\rangle$ to $|2\rangle$. After about 15 ms the initial state is almost completely recovered.

made possible a wide variety of experiments for studying the quantum measurement problem on a fundamental level. In this paper, we have shown the first realization of the quantum Zeno effect on the runaway harmonic oscillator system—a cavity field coupled to a classical microwave source. The repetitive measurement of the field's photon number dramatically inhibits its rapid growth due to its continual projection back to vacuum. Different from the QND measurement, the weak measurement provides us with only partial information on the photon number, but still projects the field into a new state, depending on the measurement outcome. We propose to use this information and to actively increase the probability of any desired photon number by injecting into the cavity a coherent pulse of the precalculated phase and amplitude. After iterating the projection/injection process many times, it should be possible to avoid the randomness in the outcome of the full QND measurement, consisting of several tens of individual weak measurements, and to deterministically steer the cavity field towards any desired photon-number state. Currently, we are preparing the realization of the corresponding experiment.

Acknowledgments

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