DYNAMIC MOTION PLANNING OF A DISTRIBUTED COLLECTOR SOLAR FIELD

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Abstract: This paper is concerned with motion planing for distributed collector solar fields. The problem consists in selecting the time profile of the manipulated variable (oil flow) such that the state (temperature distribution along the field) is driven from one value to another as specified. The problem is solved by using the methods of flat systems and a change of the time variable. Two solutions are provided, one directly for the distributed parameter model and another for a lumped parameter model resulting from space sampling of the distributed model.

Keywords: Flat Systems, Orbital flatness, Motion Planning, Distributed Systems, Solar Energy, Distributed Collector Solar Fields

1. INTRODUCTION.

The objective of distributed collector solar fields consists in collecting energy from sun radiation and storing it in the thermal form. They are made of curved mirrors which concentrate direct incident sun light in a pipe located at their focus. Inside this pipe flows an oil able to store thermal energy. In very general terms, the control objective consists in manipulating the oil flow such that the temperature of the oil at the outlet of the pipe has some prescribed value. Although many different strategies for achieving this objective have been considered in the rich literature devoted to the subject (see (Silva *et al.*, 2003) for an up to date review as well as a more detailed description of the plant), this is mainly concerned with the regulation problem and disturbance rejection. Opposite, in this paper the problem of motion planning is considered, using the concept of flatness (Fliess *et al.*, 1995; Martin *et al.*, 2001).

The field is described by the simple hyperbolic partial differential equation (PDE):

$$\frac{\partial T(z,t)}{\partial t} + u(t)\frac{\partial T(z,t)}{\partial z} = \alpha R(t) \tag{1}$$

where $T(z,t)$ denotes the oil temperature at position z and at time t, u is the oil velocity (proportional to flow), taken as the manipulated variable, R is a known function of solar radiation and α is a parameter which is assumed to be constant and known. The length of the pipe is denoted by L. The state of this distributed parameter system is described at each time t by the function $\{T(z,t),\, 0 \leq z \leq L\}$. The dynamic planning problem consists (Lynch and Rudolph, 2002) of finding a control law (in the form of a time profile for u) driving the state between two states within the reachable set of the model.

The real solar collector field considered is obviously not exactly modelled by (1). This equation is only used to perform motion planing of the dominant dynamics, which is the subject of this paper. Tackling the actual plant requires an adaptive feedback controller which is able to compensate the model mismatches and incorporates the motion planing block (Igreja *et al.*, 2004).

Although for (1) there is probably not a flat output, it is possible to introduce a time scaling such that the

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system becomes flat (Fliess *et al.*, 1995; Guay, 1999; Respondek, 1998; Vollmer and Raisch, 2003). Such a system is said orbitally flat. Thus, the solution to the problem at hand is obtained by introducing a change of variable $\tau(t)$ such that, in the new coordinate space $(z,\,\tau)$ a flat output $\hat{y}$ is found

$$\hat{y}(\tau) = h(T(z,\,\tau),\,u,\,\dot{u},\,\ldots) \qquad (2)$$

Actually, for (1) in the transformed time τ the model becomes linear (Silva, 2003). Once the flat output is defined, it is possible to describe all trajectories $T(z,\tau)$, $u(\tau)$ satisfying the transformed PDE as a function of the flat output and its derivatives, in particular

$$T(z,\,\tau) = \varphi(\hat{y},\,\dot{\hat{y}},\,\ddot{\hat{y}},\,\ldots) \qquad (3)$$

$$u(\tau) = \chi(\hat{y},\,\dot{\hat{y}},\,\ddot{\hat{y}},\,\ldots) \qquad (4)$$

Since all linear controllable systems are flat (Martin, 2001), the fact that the transformed system is linear ensures the existence of a flat output. It should also be remarked that the system is of distributed parameter type. Flatness based control design for distributed parameter systems is a subject of growing interest (Rouchon, 2001; Rudolph *et al.*, 2003). In (Lynch, 2002) the problem is considered for quasi-linear parabolic systems. A major difference with respect to the problem considered in this paper consists in the fact that, while most works refer to boundary control, here the manipulated variable is a flow. The problem considered finds its interest in the fact that, due to its simplicity it allows a net application of the methods, thereby forming a paradigm for more complicated situations. Furthermore, it concerns an application of great practical interest.

The paper is organized as follows: After formulating the problem and highlighting the methods for its solution (this section), the approach of orbital flatness for getting a flat output for eq.(1) is presented in section 2. The flat output of the transformed model is used in section 3 to solve the dynamic planning problem for the PDE model. Section 4 mentions another approach based on the approximation of the PDE model by a lumped parameter model. Section 5 provides a numerical example and section 6 draws conclusions.

2. FLAT OUTPUT FOR THE PDE MODEL.

The strategy for obtaining a flat output for (1) is detailed hereafter.

2.1 *Changing the time scale.*

According to the approach of orbital flatness, consider the change of time scale

$$\tau(t) = \int_0^t u(\sigma)d\sigma\,;\;\; \frac{d\tau}{dt} = u(t) \qquad (5)$$

This change of variable introduces a "natural" time scale associated to oil flow which, as shown in (Silva, 2003), linearizes the plant model, forcing the characteristic lines of (1) to become straight lines. Assumptions should be made (Vollmer, 2003) that the mapping between t and τ is bijective, that τ is a monotonically function of t and that it goes to infinity if and only if t goes to infinity. The validity of these assumptions is ensured by natural physical constraints in the practical problem at hand. Under these hypothesis, "real" time t can be recovered from the transformed time τ from

$$t(\tau) = \int_0^\tau \frac{1}{u(\sigma)}d\sigma \qquad (6)$$

In the time scale τ eq. (1 becomes

$$\frac{\partial T(z,\tau)}{\partial \tau} + \frac{\partial T(z,\tau)}{\partial z} = f(\tau) \qquad (7)$$

where

$$f(\tau) \triangleq \frac{\alpha R(t(\tau))}{u(t(\tau))} \qquad (8)$$

is the manipulated variable.

The general solution of (7) is given by

$$T(z,\,\tau) = \phi(\tau - z + C) + F(\tau) \qquad (9)$$

in which $\phi(x)$ is a function to be found depending on initial conditions, satisfying the homogeneous equation ((7) with $f = 0$), $F(\tau)$ is a primitive of the transformed input $f(\tau)$ and C is a constant.

2.2 *Boundary conditions – Flat output*

Consider the boundary condition given by the gradient with respect to z, computed at $z = L$:

$$\frac{\partial T(z,\,\tau)}{\partial z}|_{z=L} = y(\tau) \qquad (10)$$

where $y(\tau)$ is a specified function and L is the total length of the pipe. The boundary condition for $z = L$ yields $\phi(x)$:

$$\frac{d\phi}{dx}|_{z=L} = -y(\tau) \qquad (11)$$

For $z = L$, a constant, it follows that

$$\frac{d\phi(\tau)}{d\tau} = -y(\tau) \qquad (12)$$

Hence, integrating and inserting in (9):

$$T(z,\,\tau) = F(\tau) - Y(\tau + L - z) + D \qquad (13)$$

where $Y(\tau)$ is a primitive of $y(\tau)$ and D is a constant.

The manipulated variable $f(\tau)$ is yielded by the boundary condition for $z = 0$:

$$T_0 = F(\tau) - Y(\tau + L) \qquad (14)$$

where T_0 is the inlet oil temperature, assumed constant. Hence

$$f(\tau) = y(\tau + L) \qquad (15)$$

and

$$T(z,\,\tau) = T_0 + Y(\tau + L) - Y(\tau + L - z) \qquad (16)$$

Developing $y(\tau+L)$, $Y(\tau+L)$ and $Y(\tau+L-z)$ in Taylor series around τ, yields:

$$f(\tau)=\sum_{k=0}^{\infty}\frac{L^k}{k!}\overset{(k)}{y}(\tau) \tag{17}$$

$$T(z,\,\tau)=T_0+\sum_{k=1}^{\infty}\frac{L^k}{k!}\overset{(k-1)}{y}(\tau)- \\ -\sum_{k=1}^{\infty}\frac{(L-z)^k}{k!}\overset{(k-1)}{y}(\tau) \tag{18}$$

Taking y (gradient of the temperature at the pipe outlet) as the flat output, the above expressions provide the algebraic expressions needed for dynamic motion planning. The trajectories depend only on the knowledge of the inlet temperature and on the successive derivatives with respect to time of the flat output.

3. DYNAMIC PLANNING FOR THE PDE MODEL.

Motion planning connects stationary states, for which

$$\frac{dT_{ss}(z)}{dz}=f_{ss} \tag{19}$$

and hence

$$T_{ss}(z)=f_{ss}z+T_0 \tag{20}$$

where f_{ss} is the gradient of the temperature with respect to space and T_{ss} is the temperature along the pipe in steady state. Planning is made (fig. 1) such that the temperatures along the pipe moves from the stationary state

$$T(z,0)=C_1z+C_{01} \tag{21}$$

with

$$T_0=C_{01};\qquad T(L,\,0)=C_1L+C_{01} \tag{22}$$

to the new stationary state

$$T(z,\tau^*)=C_2z+C_{01} \tag{23}$$

with

$$T(L,\,\tau^*)=C_2L+C_{01} \tag{24}$$

Hence

$$C_1=\frac{T(L,\,0)-T_0}{L} \tag{25}$$

$$C_2=\frac{T(L,\,\tau^*)-T_0}{L} \tag{26}$$

The transfer is performed according to a particular profile as explained below. If a sequence of way points is specified, this problem may be solved by concatenating a corresponding sequence of transitions performed by using the method described here. Furthermore, once $\tau=\tau^*$ is reached, the system is at rest because the derivatives of $y(\tau)$ vanish, a fact due to the properties of the transfer profile selected.

The trajectory connecting two stationary states at times $\tau=0$ and $\tau=\tau^*$ is defined by an exponential type Gevrey function of class α (Rudolph, 2003), $\Phi_{y\sigma}$. This function (fig. 2) provides a profile for changing the flat output, given by:

$$y(\tau)=C_1+(C_2-C_1)\Phi_{y\sigma}\left(\frac{\tau}{\tau^*}\right) \tag{27}$$

in which

$$\Phi_{y\sigma}(0)=0 \qquad \tau\leq 0 \tag{28}$$

$$\Phi_{y\sigma}(1)=1 \qquad \tau\geq\tau^* \tag{29}$$

and all the derivatives computed at 0 and τ^* being zero. This allows a smooth transition between stationary states.

The initial and final values of the manipulated variable are given by

$$f(0)=y(L)\cong C_1 \quad f(\tau^*)=y(\tau^*+L)=C_2 \tag{30}$$

Using the plant continuous time model, the trajectories are given by

$$T(z,\,\tau)=T_0+D(z,\,\tau) \tag{31}$$

$$f(\tau)=y(\tau+L) \tag{32}$$

in which

$$D(z,\,\tau)=\int_0^{\tau}\left(y(\sigma+L)-y(\sigma+L-z)\right)d\sigma \tag{33}$$

and

$$D(z,\,0)=C_1z \tag{34}$$

$$D(z,\,\tau^*)=C_2z \tag{35}$$

It is remarked that, by making

$$y(\tau)=C_1+(C_2-C_1)\Phi_{y\sigma}\left(\frac{\tau-L}{\tau^*}\right) \tag{36}$$

and

$$\Phi_{y\sigma}(0)=0 \qquad \tau\leq L \tag{37}$$

$$\Phi_{y\sigma}(1)=1 \qquad \tau\geq\tau^*+L \tag{38}$$

the advance disappears:

$$T(z,\,\tau)=T_0+\int_0^{\tau}(f(\sigma)-f(\sigma-z))d\sigma \tag{39}$$

$$f(\tau)=C_1+(C_2-C_1)\Phi_{f\sigma}\left(\frac{\tau}{\tau^*}\right) \tag{40}$$

The issue of the convergence of the series (18) is not addressed in this paper. In similar problems (Lynch, 2002) convergence is ensured by selecting the profile of the flat output transfer as a Gevrey function, as performed here. Furthermore, there is strong numerical evidence on the convergence of (18).

4. DYNAMIC PLANNING WITH THE DISCRETE MODEL.

A different approach for solving the motion planning problem consists in approximating eq. (7) by a lumped parameter state space model obtained by space sampling. This is the approach followed in (Barão*et al.*, 2002) to design an adaptive controller based on feedback linearization for eq. (1). For that sake, define a state vector formed by N temperatures

T_k measured at points kh, where $h = L/N$. The approximation of the space derivative in (7) by backward finite differences yields the linear state space model

$$\dot{x} = \frac{1}{h}Ax + Bf(\tau) \tag{41}$$

where it is assumed that $T_0 = 0$,

$$x = [T_1 \; \ldots \; T_N]^T$$

and

$$A = \begin{bmatrix} -1 & 0 & \ldots & 0 \\ 1 & -1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 1 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{42}$$

Since this is a controllable system, the flat output and the control law can easily be obtained by using standard methods (Henson and Seborg, 1997) as:

$$y = \hat{C}x \tag{43}$$

$$\hat{C} = \bar{B}W^{-1} \tag{44}$$

with

$$W = [\bar{B} \; A\bar{B} \; \ldots \; A^{N-1}\bar{B}] \tag{45}$$

and

$$\bar{B} = [0 \; \ldots \; 0 \; \beta]^T \tag{46}$$

and $\beta \neq 0$ an arbitrary constant. Selecting

$$\beta = \frac{1}{h^N} \tag{47}$$

yields

$$\hat{C} = \frac{1}{h}[0 \; \ldots \; 0 \; -1 \; 1] \tag{48}$$

and hence

$$y = \frac{1}{h}(x_N - x_{N-1}) \tag{49}$$

This confirms the result obtained in the previous section on the basis of the PDE model. The flat output is a backwards finite difference approximation of the gradient with respect to space of temperature, at the pipe outlet.

The change of variable leading to normal form is given by

$$\zeta = \Pi x \tag{50}$$

with

$$\zeta = \begin{bmatrix} y \\ \dot{y} \\ \vdots \\ \overset{(N-1)}{y} \end{bmatrix} \qquad \Pi = \begin{bmatrix} \hat{C} \\ \hat{C}A \\ \vdots \\ \hat{C}A^{N-1} \end{bmatrix} \tag{51}$$

The manipulated variable sequence which transfers the state is given by

$$f(\tau) = \frac{\overset{(N)}{y} - \hat{C}A^N x(\tau)}{\hat{C}A^{N-1}B} \tag{52}$$

Or, interms of the flat output

$$f(\tau) = \frac{\overset{(N)}{y} - \hat{C}A^N \Pi^{-1}\zeta(\tau)}{\hat{C}A^{N-1}B} \tag{53}$$

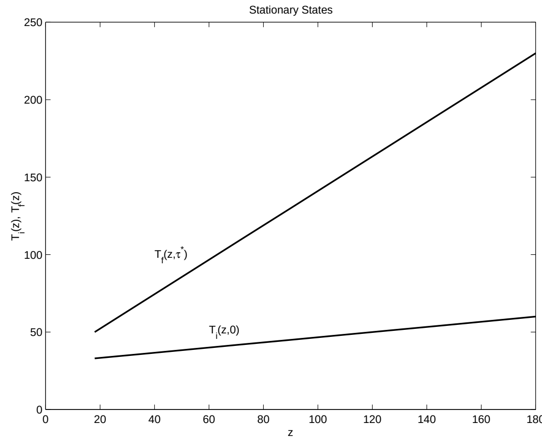

Fig. 1. Initial and final desired states.

The inclusion of inlet oil temperature can readily be made by repeating the previous procedure with

$$\bar{x} = x - \boldsymbol{T}_0 \tag{54}$$

where $\boldsymbol{T_0}$ is a column vector with the dimension of x and all its elements equal to T_0. It is then possible to show that this is consistent with the results obtained directly with the PDE in the previous section.

Furthermore, it is also possible to show that

$$\lim_{N\to\infty} \overset{(k)}{y} = \lim_{N\to\infty} \frac{(-1)^{k-1}}{h^k} \sum_{i=0}^{k} \begin{pmatrix} k \\ i \end{pmatrix} (-1)^i T_{N-i} =$$

$$= (-1)^{k-1} \frac{\partial^k T(z,\,\tau)}{\partial z^k}|_{z=L} \tag{55}$$

$$\lim_{N\to\infty} f = \lim_{N\to\infty} \sum_{k=0}^{N} \begin{pmatrix} N \\ k \end{pmatrix} h^k \overset{(k)}{y} = y(\tau + L) \tag{56}$$

which again is consistent with the PDE version.

5. EXAMPLE.

An example of dynamic motion control for a distributed collector solar field is presented in figs. 1-8. Hereafter, z and τ are expressed in $[m]$ and temperature in $[^oC]$. Initial and final desired states are shown in fig. 1. These are linear functions of space which, as is easily shown, correspond to the equilibrium states of (1). Fig. 2 shows the Gevrey function for the flat output in the transformed time scale τ. As explained, it provides the profile along which the flat output is changed in time. Fig. 4 shows the manipulated variable in transformed time as yielded by the design procedure, and fig. 3 the corresponding temperatures for a few points along the pipe. Having solved the problem in the time scale τ, the time transform has to be inverted. The relation between transformed time and real time is shown in fig. 5, and figs. 7 and 6 correspond to the desired profiles of manipulated variable and state, obtained by changing the time scale in figs. 3 and 4. Finally, fig. 8 provides an overall view of state transfer.

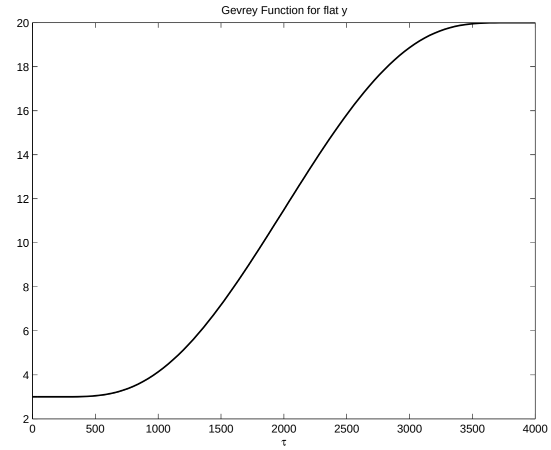

Fig. 2. Gevrey function for the flat output in the transformed time scale.

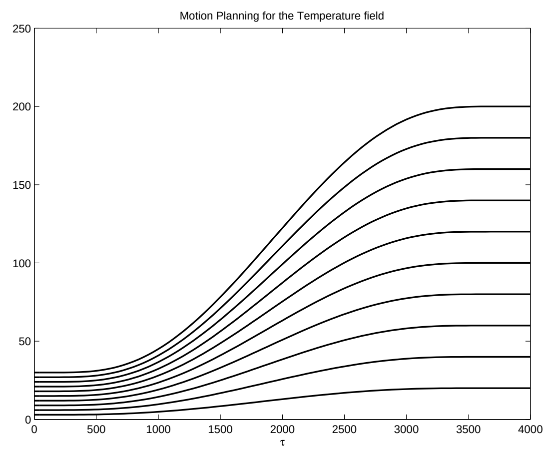

Fig. 3. Motion planning for temperatures inside the pipe, in the transformed time scale.

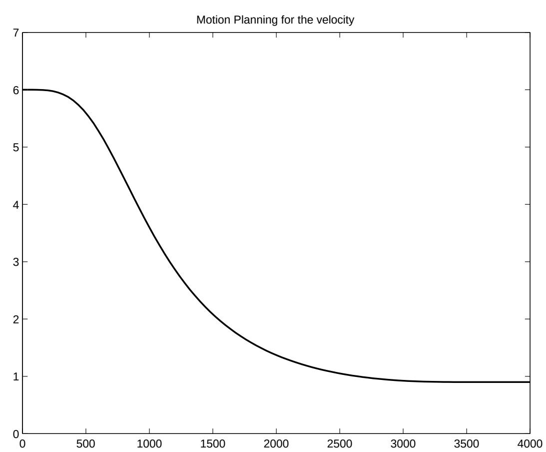

Fig. 4. Motion planning for the oil velocity (manipulated variable) in the transformed time scale.

6. CONCLUSIONS.

The problem of dynamic motion planning for a distributed collector solar system has been considered and solved using the methods of orbital flatness. The method can be applied to other similar systems involving transport phenomena, such as moisture control.

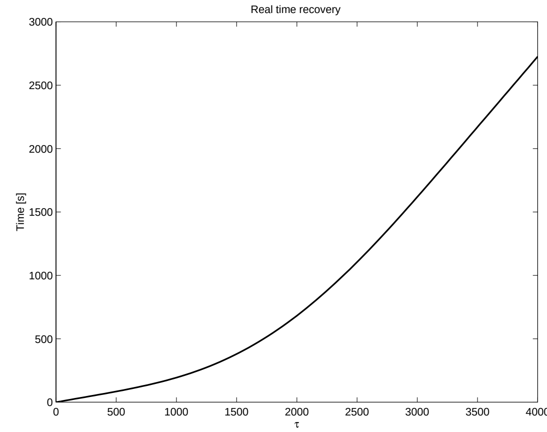

Fig. 5. Relation between the real time and the transformed time τ.

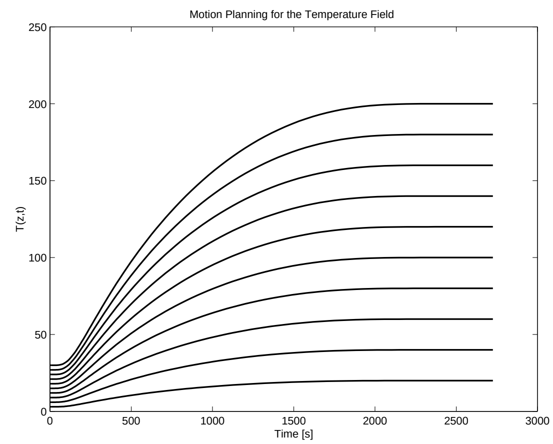

Fig. 6. Motion planning for temperatures inside the pipe, in the real time scale

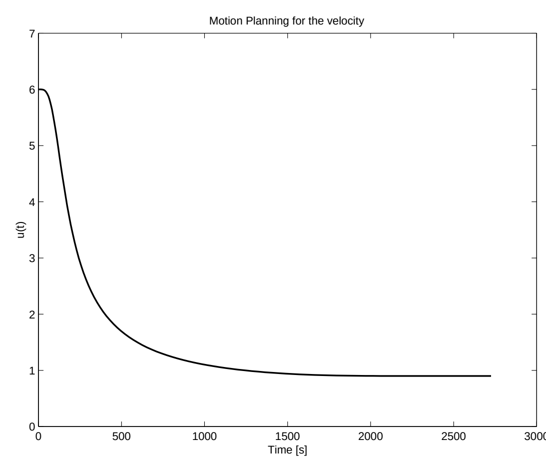

Fig. 7. Motion planning for the oil velocity in the real time scale.

Furthermore, due to its simplicity, the problem considered allows a net application of the methods, thereby forming a paradigm for more complicated situations.

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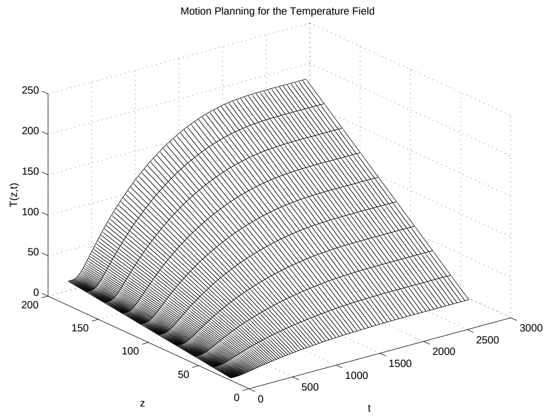

Fig. 8. State transfer in the real time scale.

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