Theory and practice in the motion planning and control of a flexible robot arm using Mikusiński operators

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Abstract

The motion planning and the synthesis of a tracking controller of a flexible robot arm is studied in an algebraic framework using Mikusiński's operational calculus. Experimental results are reported.

Keywords

Flexible robot arm, infinite dimensional systems, rings, modules, Mikusiński's operators, Euler–Bernoulli partial differential equation.

1 Introduction

Vibration control of flexible robots is getting some importance in space applications and in the optimization of the weight/power ratio and the robot dynamics. Control laws are derived most often on the basis of a discrete flexible model (*see, e.g.,* [10, 8] or [3]) However, several authors, such as [11], use partial differential integral equations or dynamic models of rigid robots [2] in the control of flexible robot arms.

In a complementary spirit, we present here techniques for moving such an arm from rest to rest while specifying the motion of a particular point. The proposed techniques lead to exact and explicit formulae, *i.e.* they allow the solution without integration of any differential equation. These formulae are seen to result in computationally efficient and robust schemes. The experimental experience confirms the practical relevance of the theoretical predictions.

The elementary foundations of an algebraic theory of constant linear systems are laid down, which, generalizing [13, 15, 26], leads to the consideration of finite type modules over domains. The notions of controllability and observability are defined in this context. To deal with equations such as the Euler–Bernoulli one, the ground ring is generated by *Mikusiński's operators* [23, 24], or, more precisely, by *operational functions* [23, 24]; recall that the operational calculus of Mikusiński [23, 24] forms a substitute as elegant as easy to handle for the Laplace transformation. The freeness of a certain module, the properties of which are obtained through homological arguments (*see* [12, 19])[1], permits to obtain the desired behavior of the beam with an open loop control, by assigning a trajectory to a basis of this module: this is an approach analogous to the *flatness based control* [14] of nonlinear finite dimensional systems. A regulation with passivity, close to the proportional–derivative one used in [1], leads to the stabilization. The numerical computations use series developments of Mikusiński's operators [24] and a result on Gevrey-Roumieu functions due to Ramis [29].

Comparisons with the very rich literature on distributed parameter systems (*see, e.g.,* [21, 22, 4, 9, 18] and the references therein)[2] will be made elsewhere. Notice, however, that such a comparison has been made in the thesis of one of the authors [26] (*see also* [15, 16, 25, 27]) for another class of infinite dimensional systems, viz.

[1] We especially use the resolution of Serre's conjecture by Quillen [28] and Suslin [32], already exploited in [26] (*see also* [15, 16]).

[2] We refer the reader also to also to [5, 7] for the consideration of equations of the same type as ours.

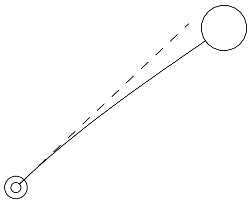


Figure 1: *Rotating beam with an end mass.*

the one of localized delay systems.

2 The flexible robot arm

The flexible robot arm under consideration consists in a flexible beam with length L. Its end $r = 0$ is clamped in the axle of a motor, the angle of which with respect to a fixed reference is θ, a mass M being attached at the other end $r = L$. The movement of this beam is supposed to be described by the following model — which is valid under standard hypothesis like linear elasticity, small deflection, and negligible Coriolis forces, *i.e.* $\dot{\theta}$ small — and boundary conditions:

$$\begin{aligned}
\frac{\partial^4 V}{\partial^4 r}(r,\tau) &= \frac{-\rho S}{EI}\frac{\partial^2}{\partial \tau^2}[V(r,\tau) + r\theta(\tau)] \\
J_m \frac{d^2\theta}{d\tau^2}(\tau) &= \mathcal{T} + EI\frac{\partial^2 V}{\partial r^2}(0,\tau) \\
V(0,\tau) &= 0, \qquad \frac{\partial V}{\partial r}(0,\tau) = 0 \\
\frac{\partial^2}{\partial r^2}V(L,\tau) &= \frac{-J}{EI}\frac{\partial^2}{\partial \tau^2}\left[\frac{\partial V}{\partial r}(L,\tau) + \theta(\tau)\right] \\
\frac{\partial^3 V}{\partial r^3}(L,\tau) &= \frac{M}{EI}\frac{\partial^2}{\partial \tau^2}\left[V(L,\tau) + L\theta(\tau)\right],
\end{aligned} \tag{1}$$

where $V(r,\tau)$ is the field of deflections of the beam with respect to its equilibrium, which is determined by the angle θ of the rotating axis. Here $\mathcal{T}$ is the control torque, and EI, ρ, J, J_m, and S are the physical constants in a common notation.

3 R-linear systems

The controllability notions introduced hereafter are adapted from [13, 15, 16, 26]. The algebraic language is elementary and can be found in several textbooks, such as, *e.g.* , [12, 19].

Let R be a commutative ring with unity and with no zero divisors. An *R-linear system*, or *R-system*, Σ is a finitely generated R–module Λ. Denote as $[\xi]$ the sub-R–module generated by a family ξ of Λ. An element w of an R-module M is said to be torsion if it satisfies a relation of the form $pw = 0$ with $p \in M$, $p \neq 0$.

Example 1. – A relationship between the previous module theoretic definition and the system equations can be seen on the following example.

An *input* $\boldsymbol{u} = (u_1, \ldots, u_m)$ is a finite sequence of elements in Λ such that the quotient module $\Lambda/[\boldsymbol{u}]$ is torsion; $\boldsymbol{u}$ is called *independent* if $[\boldsymbol{u}]$ is a free R–module of rank m. An *output* $\boldsymbol{y} = (y_1, \ldots, y_p)$ is a finite sequence of elements in Λ.

Example 2. – Take $R = k[\frac{d}{dt}]$, where k is a commutative field, such as $\mathbb{R}$. One then obtains the module-theoretic approach to finite dimensional linear systems (*see* [13]).

Example 3. – Set $R = \mathbb{R}[\frac{d}{dt}, \delta_1, \ldots, \delta_r]$, where δ_i, $i = 1, \ldots, r$, is a localized delay operator: for every function $f : \mathbb{R} \to \mathbb{R}$, $\delta_i f(t) = f(t - h_i)$, $h_i \in \mathbb{R}_+$. One thus obtains the constant linear systems with localized delays (*see* [26] and also [15, 16]).

Let A be an R-algebra. The R-system Σ is called *A-torsion-free* (resp. *projective, free*) *controllable* if the A–module $A \otimes_R \Lambda$ is torsion free (resp. projective, free). The A-free (resp. projective) controllablility implies the A-projective (resp. torsion free) controllability.

Remark 4. – An R-system is A-torsion free controllable if no variable of the system satisfies an autonomous equation with coefficients in A (autonomous meaning not involving the input). If $A = k[\frac{d}{dt}]$, this equation is a differential one; if $A = \mathbb{R}[\frac{d}{dt}, \delta_1, \ldots, \delta_r]$, it is a difference-differential one.
An R-system Σ is A-free controllable if there exists a basis $b_1, \ldots, b_m$, *i.e.* , if the b_is are linearily A-independent and generate Λ.

The following result is adapted from [31]:

Theorem and Definition 1 *Assume Σ is R-torsion-free controllable. Then there exists $\pi \in R, \pi \neq 0$, such that $\Lambda_\pi = R[\pi^{-1}] \otimes_R \Lambda$ is free. The system Σ is called* π-free.

Example 5. – The type of freeness in the preceding notion amounts to allowing the formal inversion of the operator π in all algebraic calculations. For example, considering the delay system $\dot{y}(t) = u(t-1)$, the corresponding module Λ over $\mathbb{R}[\frac{d}{dt}, \delta]$ has the equation $\frac{d}{dt}y = \delta u$. It is δ-free with basis y: one has $u = \delta^{-1}y$, or $u(t) = y(t+1)$.

Remark 6. – Unlike in [15, 26, 13], we do not introduce a state representation for Σ. It seems indeed being useless for the present applications.

4 Mikusiński's operators

The continuous functions $[0, +\infty[\to \mathbb{C}$, together with addition and convolution product, form a commutative ring, with no zero divisors, as shown by a famous theorem due to Titchmarsh. Its field of fractions $\mathcal{M}$ is the set of *Mikusiński's operators* [23, 24]. The unit of $\mathcal{M}$ is the *Dirac operator*. The *derivation operator* is denoted as s.

4.1 Mikusiński systems

A *linear system over Mikusiński's operators*, or, more briefly, a *Mikusiński system*, is an R-linear system, where $R \subset \mathcal{M}$ is a finite type k-algebra, with k a subfield of $\mathbb{C}$.

Example 7. – With $R = k[s, e^{-h_1 s}, \ldots, e^{-h_r s}]$, $h_1, \ldots, h_r \in \mathbb{R}_+$, where $e^{-h_1 s}, \ldots, e^{-h_r s}$ are *shift* operators (*cf.* [23, 24]), we obtain systems with localized delays.

Example 8. – With R generated by s and a finite number of Mikusiński operators, we obtain linear systems with generalized localized or distributed delays.

4.2 Parametric Mikusiński systems

Consider the ring R_I of *operational functions* (*cf.* [23, 24]) $I \to \mathcal{M}$, where I is an interval of $\mathbb{R}$. A *linear parametric system over the operators of Mikusiński*, or, more briefly, a *parametric system of Mikusiński* is a module of finite type over a k-algebra $R \subset R_I$ with no zero divisors and of finite type. A natural example is given below.

5 The flexible robot arm as a parametric Mikusiński system

Set

$$\alpha = \sqrt{\frac{\rho S}{EI}} L^2, \quad r = Lx, \quad \tau = \alpha t, \quad w = V + r\theta.$$

With this, the equations (1) are transformed into the well-known Euler-Bernoulli partial differential equation together with the boundary conditions below.

$$\frac{\partial^2 w}{\partial t^2} = -\frac{\partial^4 w}{\partial^4 x} \tag{2}$$

$$w(0,t) = 0, \qquad \frac{\partial w(0,t)}{\partial x} = L\theta(t) \tag{3}$$

$$\frac{\partial^2 w(1,t)}{\partial x^2} = -\lambda \frac{\partial^3 w}{\partial x^2 \partial t}(1,t), \tag{4}$$

$$\frac{\partial^3 w(1,t)}{\partial x^3} = \mu \frac{\partial^2 w}{\partial t^2}(1,t), \tag{5}$$

with

$$\lambda = \frac{J}{\rho S L^3}, \qquad \mu = \frac{M}{\rho S L}.$$

5.1 Modules

With the initial conditions $w(x,0) = 0$ and $\frac{\partial w(x,0)}{\partial t} = 0$, Mikusiński's operational calculus [23, 24] associates (2) with the s-dependent ordinary differential equation $s^2 \hat{w} = -\hat{w}^{(4)}$, where $\hat{w}$ denotes the operational function corresponding to w. The operators $\sqrt{s}$ and $i\sqrt{s}$ being *logarithmic* [23, 24], the general solution of (2) can be expressed as

$$\hat{w}(x,s) = a(s)C_x^+(s) + b(s)C_x^-(s) + c(s)S_x^+(s) + d(s)S_x^-(s) \tag{6}$$

with

$$C_x^+(s) = \frac{C_x(s) + \overline{C_x(s)}}{2} \quad C_x^-(s) = \frac{C_x(s) - \overline{C_x(s)}}{2i}$$

$$S_x^+(s) = \frac{\overline{S^x(s)} + iS_x(s)}{2h\sqrt{s}} \quad S_x^-(s) = \frac{-\overline{S_x(s)} + iS_x(s)}{2\bar{h}\sqrt{s}}$$

and

$$C_x(s) = \cosh[h\sqrt{s}(1-x)] \qquad S_x(s) = \sinh[h\sqrt{s}(1-x)],$$

where $h = e^{i\pi/4}$ (i being $\sqrt{-1}$). The coefficients a, b, c, d are determined by the boundary conditions:

$$\begin{aligned} aC_0^+ + bC_0^- + cS_0^+ + dS_0^- &= 0 \\ s(-aS_0^+ + bS_0^-) + cC_0^- + dC_0^+ &= \hat{u} \\ b &= -\lambda s d \\ c &= \mu s a \end{aligned} \tag{7}$$

Set

$$R_x = \mathbb{C}\left[s, C_x^+, C_x^-, S_x^+, S_x^-\right]$$

and let M be the R_0–module generated by a, b, c, d, and $\hat{u}$.

Lemma 1 *M is of rank 1, torsion free, but not free.*

Proof

The first two assumptions are easily shown. For the third one, consider the following presentation matrix of M

$$\begin{pmatrix} C_0^+ & C_0^- & S_0^+ & S_0^- & 0 \\ -sS_0^+ & sS_0^- & C_0^- & C_0^+ & -1 \\ 0 & 1 & 0 & \lambda s & 0 \\ -\mu s & 0 & 1 & 0 & 0 \end{pmatrix}$$

Setting $z_1 = C_x^+$, $z_2 = C_x^-$, $z_3 = S_x^+$, $z_4 = S_x^-$, the minors of order four are zero for $z_1 = z_2 = z_3 = z_4 = 0$. A lemma of [6] and the resolution of Serre's conjecture in [28, 32] imply that M is not free. □

Set

$$\omega_x = C_x^+ + \mu s S_x^+ \qquad \pi_x = \lambda s C_x^- - S_x^-.$$

We have:

Lemma 2 *The module $R_0[\pi_0^{-1}] \otimes_R M$ is free with basis $\hat{y} = {\pi_0}^{-1} a$.*

Proof

One readily obtains: $a = \pi_0 \hat{y}$, $b = -\lambda s \omega_0 \hat{y}$, $c = \mu s \pi_0 \hat{y}$, and $d = \omega_0 \hat{y}$. □

Then

$$\hat{u} = \left[(-sS_0^+ + \mu s C_0^-)\pi_0 + (C_0^+ - \lambda s^2 S_0^-)\omega_0)\right] \hat{y}. \tag{8}$$

Note that (6) can be rewritten as

$$\hat{w}(x,s) = (\omega_x \pi_0 + \pi_x \omega_0)\hat{y}. \tag{9}$$

It follows:

Theorem 1 *The R_x–module Λ^x generated by $\hat{u}$ and $\hat{w}$ is torsion free, of rank 1. The localized $R_x[\pi_0^{-1}]$–module $\Lambda^x_{\pi_0}$, generated by $\hat{u}$ and $\hat{w}$, is free, with basis $\hat{y}$.*

Proof

Indeed $\hat{w}(1,s)=\pi_0\hat{y}=a$ and $\Lambda^x_{\pi_0}$ thus contains $\hat{y}$. The result then follows from (8) and (9). □

In other terms, one has R_x–torsion free controllability (resp. $R_x[\pi_0^{-1}]$–free controllability) for the parametric Mikusiński R_x–system (resp. $R_x[\pi_0^{-1}]$–system), associated with Λ^x (resp. $\Lambda^x_{\pi_0}$).

Remark 9. – Compared with flatness [14], already mentioned in the introduction, there exists a great variety of possible choices for the module, and thus also for a basis. Indeed, Mikusiński's operators belong to a field, $\mathcal{M}$. The selection should be done in view of simplifications of the calculations (convergent series) and of the physical meaning.

5.2 Trajectory and open loop control calculation

Developping formulae (8) and (9) leads to the consideration of operators like $\sum_{n\geq 0}\alpha_n s^n$. For the treatment of these series, we need the following considerations. A C^∞ function $f: I\to\mathbb{R}$, where I is an open interval of $\mathbb{R}$, is of class $\mathcal{C}\{\Gamma(\mu n)\}$ (*see* [24]), if, and only if, there exists $M_f, R_f\in\mathbb{R}_+$ such that, for all $t\in I$ and for all derivation order n,

$$|f^{(n)}(t)|\leq M_f\Gamma(\mu n)(R_f)^n,$$

where Γ is the Euler function. In other words, the functions of class $\mathcal{C}\{\Gamma(\mu n)\}$ are regular functions f such that the series $\sum_{n\geq 0} f^{(n)}(t)X^n$ is Gevrey of order μ (*see* [29])[3] uniformly in t. It is clear that analytic functions belong to $\mathcal{C}\{\Gamma(\mu n)\}$, where $\mu\geq 1$. For $\mu>1$, the class $\mathcal{C}\{\Gamma(\mu n)\}$ is much bigger, by a theorem due to Denjoy-Carleman (*see* [24]): it includes, in particular, sigmoid functions of the type partition of unity (*see* [24, example of page 125, with $\varepsilon_n=(\Gamma(\mu n))^{-1/n}$] or the function g here below). Finally, the set of functions of the class $\mathcal{C}\{\Gamma(\mu n)\}$ forms a ring for the usual addition and product. This type of function spaces is also considered in [17], where they are called *S type spaces.*

If $y(t)$ is of class $\mathcal{C}\{\Gamma(\mu n)\}$, with $\mu<2$, the following series, which corresponds to $\cosh(\sqrt{2s})y$, is absolutely convergent

$$\sum_{n\geq 0}\frac{2^n}{(2n)!}y^{(n)}(t).$$

For any specialization of the basis y to a function of class $\mathcal{C}\{\Gamma(\mu n)\}, \mu<2$, equation (8) yields

$$\begin{aligned}u(t)=&\frac{-J_m}{L\alpha^2}\left[1+\sum_{n=0}^{\infty}\frac{2^{2n+1}}{(4n+4)!}\Big((1+\lambda\mu)\frac{d^2}{dt^2}+\right.\\&\left.(4n+4)(\mu+\frac{4n+3}{2}\lambda)\Big)\frac{d^{2n+4}}{dt^{2n+4}}\right]y(t)+\\&\frac{EI}{L^2}\left[\sum_{n=0}^{\infty}\frac{2^{2n+1}}{(4n+4)!}\Big((4n+4)\Big(\frac{1}{2}+\frac{\lambda\mu}{2}\Big)\frac{d^2}{dt^2}+\right.\\&\left.(4n+3)(\mu+\frac{(4n+1)(4n+2)}{2}\lambda)\Big)\frac{d^{2n+2}}{dt^{2n+2}}\right]y(\tau)\end{aligned}\quad(10)$$

and with (9),

$$\begin{aligned}w(x,t)=&\left[\sum_{n=0}^{\infty}\frac{(-1)^n s^{2n}}{(4n)!}\left(\frac{x^{4n+1}}{2(4n+1)}+\right.\right.\\&\left.\left.\frac{(\Im-\Re)(1+i-x)^{4n+1}}{2(4n+1)}+\mu\Im(1+i-x)^{4n}\right)\right]y(t)+\\&\left[\sum_{n=0}^{\infty}\frac{(-1)^n s^{2n+2}}{(4n+4)!}\left(\frac{\lambda\mu}{2}+\right.\right.\\&\frac{(4n+2)!}{(4n+4)!}\left[(\Im-\Re)(1+i-x)^{4n+1}-x^{4n+1}\right]\\&\left.\left.-\lambda(4n+3)(4n+4)\Re(1+i-x)^{4n+2}\right)\right]y(t)\end{aligned}\quad(11)$$

where $\Re$ (resp. $\Im$) denotes the real (resp. imaginary) part. In other words, the two relations above define a family of trajectories for the hybrid system (2) parametrized by all functions of class $\mathcal{C}\{\Gamma(\mu n)\}$, $\mu<2$.

5.3 The experiment

Consider now three equilibrium states of the beam corresponding to three angles θ_1, θ_2, and θ_3. Let the reference be

$$y_r(t)=(S*g)(t)$$

with S a suitably chosen interpolating B-spline of order 6 and $[0,T]\ni t\mapsto g(t)$ an approximation of the Dirac distribution of class $\mathcal{C}\{\Gamma(\mu n)\}$. The above formulae allow to achieve $y(0)=\theta_1$, $y(T/2)=\theta_2$, and $y(T)=\theta_3$ with $y^{(n)}(0)=y^{(n)}(T/2)=y^{(n)}(T)=0$ for all $n>0$ by means of a regular control $[0,T]\ni t\mapsto u(t)$. This control law corresponds to a rotation of angle $\theta_3-\theta_1$ during the time T with a rest at θ_2 around $T/2$. The whole scheme ensures the absence of vibrations at the end of the movement $t\geq T$. The Figures 2 and 3 correspond to a beam of length $L=1.005$m with an end mass of $M=5.9$kg ($EI=47.25$N.m^2, $\rho S=2.04$kg.m^{-1}, $J=0.047$kg.m^2, $J_m=0.0018$kg.m^2). The arm is equipped with an optical sensor measuring the angle θ and with two deformation sensors (tensometrical sensors) allowing the evaluation of the deflection $w(1,t)$. The motor has a nominal torque of 3.5N. The movement of 3rad is achieved in 30s ($\theta_1=0$ and $\theta_3=3$ with a rest at $\theta_2=1.5$).

[3]See [30] for an excellent servey on divergent series.

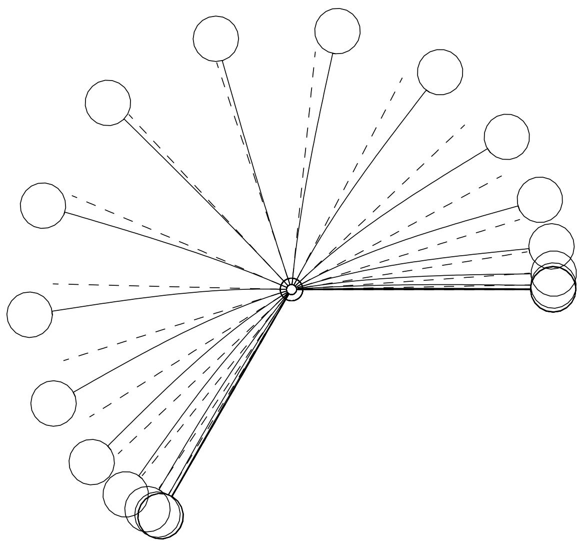


Figure 2: *Various positions of the beam during the motion.*

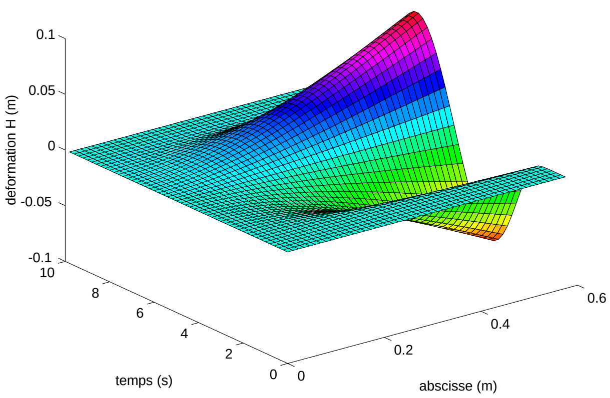


Figure 3: *The field of deflections* $w(x,t)$.

Using Proposition 5 of [29] characterizing the Gevrey-Roumieu functions of type $s > 0$, it is easy to show that the function g defined by

$$]0,1[\ni t \mapsto \exp[-(t(1-t))^{-\nu}]$$

$\nu > 0$, is of class $\mathcal{C}\{\Gamma(\mu n)\}$, $\mu = 1 + 1/\nu$. The above function y is thus of class $\mathcal{C}\{\Gamma((\mu n)\}$, with $\mu = 1.9 < 2$. The series are calculated with about ten terms.

Remark 10. – A first robustness analysis has been conducted by varying several parameters. In the following table the maximum error on the control torque for given errors of the parameters are reported.

Par.	Nominal val.	Error val.	Par. err.	Torque err.
L	1.005 m	1.1 m	10%	34%
M	5.9 kg	7.1 kg	20%	12%
ρS	2.04 kg.m^{-1}	4 kg.m^{-1}	100%	6%
EI	47.25 N.m^2	23 N.m^2	100%	1.7%
J	0.047 kg.m^2	0.094 kg.m^2	100%	5.6%

Note that the only sensitive parameters are L and M, which are always accurately measured in practice. The errors due to other usually less well known parameters, like EI, are negligible. This inherent robustness of our control scheme is to a great extent due to the exactness of the calculations. The only approximation is the truncation of the derivation operators of infinite order; this approximation appears at the very end of the calculations, though, which results at the same time in increased accuracy and robustness.

5.4 Stabilizing feedback

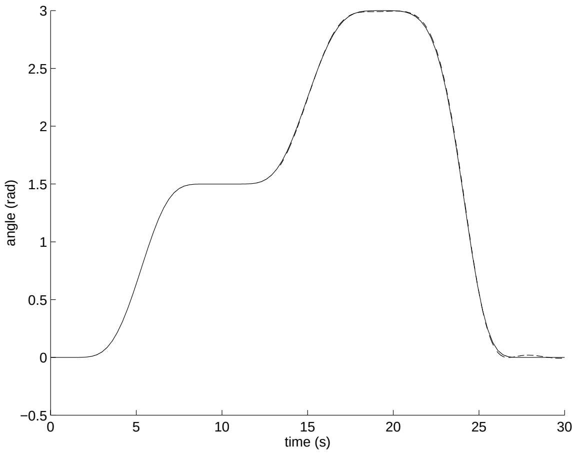
In order to follow the preceding open loop trajectories with asymptotic stability, let us introduce a passivity based feedback. The reference trajectory for θ and w are denoted by $\theta_r(\tau)$ and $w_r(x, \tau/\alpha)$. The control is the motor torque $\mathcal{T}$ with reference $\mathcal{T}_r(\tau) = -EI/L^2 \frac{\partial^2 w_r}{\partial x^2}(0, \tau/\alpha)$. An elementary damping feedback, using only the angle θ and its velocity $\frac{d\theta}{d\tau}$,

$$\mathcal{T} = \mathcal{T}_r(\tau) - k(\theta - \theta_r(\tau)) - \sigma \left(\frac{d\theta}{d\tau} - \frac{d\theta_r}{d\tau}(\tau) \right),$$

with k and σ two positive parameters, ensures the asymptotic tracking of the reference (*see* [20] for a possible choice of these gains). The figures 4, 5, and 6 show the calculated and measured angle trajectory, the tracking error, and the calculated and measured torques.

6 Conclusion

The control law is quite robust with the partially uncertain model parameters of the experimental robot. A noticable advantage of this method is the ability to freely design a nominal physical trajectory avoiding the excitation of flexible modes at the end of the movement. In



3
2.5
2
1.5
1
0.5
0
−0.5
angle (rad)
0
5
10
15
20
25
30
time (s)