Robust stabilization of flat and chained systems

M.K. BENNANI* P. ROUCHON †

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Abstract

A design method for robust stabilization of flat systems, feedback equivalent to chained ones, is proposed. The method is based on iterations of well chosen open-loop steering controls. Robustness is characterized by exponential convergence to the equilibrium for any driftless systems close to the original one. The case of chained systems of dimension 4 is treated in details. Simulation of a car-like robot are given.

Key words: chained systems, flatness, exponential stabilization, robustness, mobile robots.

1 Introduction

After the results of Coron [5, 6, 7] showing how to utilize time-varying feedback for stabilizing nonlinear plants, the practical design of such stabilizing laws is now giving rise to a rapidly growing literature.

Many papers deal with nonholonomic control systems and with the special subclass of systems in chained form with two controls (u_1, u_2) (see, e.g., [2, 4, 11, 14, 15, 19]):

$$\begin{array}{lll} \dot{x}_1 = u_1, & \dot{x}_2 = u_2, & \dot{x}_3 = u_1 x_2, \\ \dot{x}_4 = u_1 x_3, & \ldots, & \dot{x}_n = u_1 x_{n-1}. \end{array} \quad (1)$$

It appears since the work of Murray and Sastry [10], that many nonholonomic mechanical systems (the standard n-trailers systems) are feedback equivalent to such chained systems.

These chained systems are a particular subclass of flat systems [8]: $y = (x_1, x_n)$ is an obvious flat output; the entire state x with the control are function of y and its derivatives up to order $n-1$. This leads to simple motion planning algorithms for such chained systems [12, 13].

Here, we exploit this explicit trajectory parameterization in order to design robust stabilizing control scheme for driftless systems that are feedback equivalent to (1). By robust, we mean that the control not only stabilizes the original system but also exponentially stabilizes any close driftless systems obtained via smooth and small deformations of the two vector fields defining the system. As far as we know, we propose here the first stabilizing method that ensure robustness with respect to vector fields deformations representing modeling errors and parameters uncertainties. Simulations demonstrate that such control scheme are easy to compute and can be used for practical and robust stabilization of nonholonomic systems.

The basic idea of our stabilizing strategy is very simple and, as in [18, 16, 17, 3], uses a kind of dicrete-time feedback. Consider a control system described by some equations Σ_0. Denote by $[0, T] \ni t \mapsto \mathcal{U}(x^0, t)$ a smooth open-loop control that steers, for the nominal system Σ_0, the state x^0 at $t = 0$ with $u = 0$ to the state 0 at $t = T > 0$ with $u = 0$. If the real system Σ_ε (ε is a small parameter representing modeling uncertainties) differs slightly from Σ_0, then the open-loop control $\mathcal{U}$ leads to a final state that is, in general, different from 0 but close to 0. Denote by $P_\varepsilon(x^0)$ this final state. We have, by construction, $P_0(x^0) = 0$. The map P_ε can be seen as a "Poincaré"

*École des Mines de Paris, 60, Bd Saint-Michel, 75272 Paris Cedex 06.

†Author to whom correspondence should be addressed. Centre Automatique et Systèmes, École des Mines de Paris 60, Bd Saint-Michel, 75272 Paris Cedex 06, France. Tel: 33 (1) 40 51 91 15. E-mail: rouchon@cas.ensmp.fr

map: if $P_\varepsilon(0)=0$ and if P_ε is strictly contracting around 0, when successive uses of the open-loop control $\mathcal{U}$ on the perturbed system Σ_ε leads to successive states x^0, $P_\varepsilon(x^0)$, $P_\varepsilon(P_\varepsilon(x^0))=P_\varepsilon^2(x^0)$, ... converging to 0.

The contribution of the paper is the following: for systems Σ_0 that are feedback equivalent to (1), we explicitly construct open-loop controls $[0,T]\ni t\mapsto \mathcal{U}(x^0,t)$ such that the Poincaré map P_ε, associated to any driftless system Σ_ε close to Σ_0, is smooth everywhere excepted in 0 and is a strict contraction around 0. The dependence of $\mathcal{U}(x^0,t)$ with respect to t can be chosen arbitrary smooth. On the contrary the dependence of $\mathcal{U}(x^0,t)$ with respect to x^0 is smooth everywhere excepted in 0 where it is continuous with Hölder exponents strictly less than 1. Due to space limitation, we present here the method in space dimension 4. This case is enough rich to catch the generality of the method and the main steps of the proof for arbitrary dimension (see [1] for higher dimension).

The paper is organized as follows. Section 2 is devoted to chained system of dimension 4 where the proofs are given in details. Section 3 shows, for a car-like robot, simulations of the stabilizing control elaborated in section 2.

A preliminary version of this work can be found in [1], the report of the "stage de fin d'étude" of M.K. Bennani at "École Polytechnique", "promotion X91".

2 Chained systems of dimension 4

The theorem here below ensures, for chained system Σ_0 of dimension 4, the contraction of the Poincaré map P_ε around 0 when the open-loop control $\mathcal{U}$ satisfies some conditions. These conditions are verified by controls $\mathcal{U}$ explicitly given in the proposition after the theorem.

Theorem *Take $T>0$. Assume that, for all $x^0\in\mathbb{R}^4$, there exists an open-loop control $[0,T]\ni t\mapsto \mathcal{U}(x^0,t)\in\mathbb{R}^2$ steering the chained system Σ_0 (system (1) with $n=4$) from $x=x^0$ at $t=0$ with $u=0$, to $x=0$ at $t=T$ with $u=0$. Assume also that, forall $t\in[0,T]$,*

$$\begin{array}{rcl} |\mathcal{U}_1(t,x_0)| & \leq & k_1(|x_1^0|+|x_2^0|+|x_3^0|^{\alpha_3}+|x_4^0|^{\alpha_4}) \\ |\mathcal{U}_2(t,x_0)| & \leq & k_2(|x_1^0|+|x_2^0|+|x_3^0|^{1-\alpha_3}+|x_4^0|^{1-2\alpha_4}) \end{array} \tag{2}$$

where k_1, k_2, α_3, α_4 are constant independent of x^0 and t, satisfying

$$\begin{array}{l} 0<\alpha_3<1, \qquad 0<\alpha_4<1/2 \\ \min(\alpha_3,\alpha_4)+2\min(1-\alpha_3,1-2\alpha_4)\geq 1 \\ 3\min(\alpha_3,\alpha_4)+\min(1-\alpha_3,1-2\alpha_4)\geq 1. \end{array} \tag{3}$$

Assume also that the perturbed system Σ_ε is defined by

$$(\Sigma_\varepsilon)\quad \begin{array}{rcl} \dot{x_1} & = & u_1+\varepsilon(f_1(x,\varepsilon)u_1+g_1(x,\varepsilon)u_2) \\ \dot{x_2} & = & u_2+\varepsilon(f_2(x,\varepsilon)u_1+g_2(x,\varepsilon)u_2) \\ \dot{x_3} & = & u_1x_2+\varepsilon(f_3(x;\varepsilon)u_1+g_3(x,\varepsilon)u_2) \\ \dot{x_4} & = & u_1x_3+\varepsilon(f_4(x,\varepsilon)u_1+g_4(x,\varepsilon)u_2) \end{array}$$

where the f_i's and g_i's are smooth functions.

Then, for all $M>0$, there exist $C>0$ and $\eta>0$ such that, if $\|x^0\|\leq M$ and $|\varepsilon|\leq\eta$ then $\|P_\varepsilon(x^0)\|\leq \varepsilon C\,\|x^0\|$, where $P_\varepsilon(x^0)=x(T)$ with $[0,T]\ni t\mapsto x(t)$, the trajectory of Σ_ε with $u(t)=\mathcal{U}(x^0,t)$ and $x(0)=x^0$. The quantities C and η depend on M and on the maximum, over a bounded domain of $\mathbb{R}^4$ depending on M, of the f_i's and g_i's with a finite number of their x-derivatives.

Simulations seems to indicate that the constraints on the exponent α_3 and α_4 are optimal [1]

Notice that a direct analysis via standard first order variations is not enough to prove this theorem: the dependence of $\mathcal{U}$ with respect to x^0 is not Lipschitz around 0. Notice also that it is impossible to construct a steering control $\mathcal{U}(x^0,t)$ depending smoothly on t, Lipschitz in x^0 and satisfying $\mathcal{U}(x^0,0)=\mathcal{U}(x^0,T)=0$ and $\mathcal{U}(0,t)=0$, for all $t\in[0,T]$.

Proof Denote by $[0,T]\ni t\mapsto \bar{x}(t)$ the trajectory of Σ_0 with $u(t)=\mathcal{U}(x^0,t)$ and $x(0)=x^0$. By assumption, $\bar{x}(T)=0$. The integration over $[0,T]$ of the first two equations of Σ_ε leads to

$$\begin{array}{rcl} x_1(T) & = & \varepsilon\int_0^T(f_1(x,\varepsilon)u_1+g_1(x,\varepsilon)u_2)dt \\ x_2(T) & = & \varepsilon\int_0^T(f_2(x,\varepsilon)u_1+g_2(x,\varepsilon)u_2)dt \end{array} \tag{4}$$

since $\int_0^T u_1 dt = -x_1^0$ and $\int_0^T u_2 dt = -x_2^0$. For the third equation, we have,

$$\begin{aligned} x_3(T) - x_3^0 &= \int_0^T u_1 x_2\, dt + \varepsilon \int_0^T (f_3 u_1 + g_3 u_2)\, dt \\ &= [\bar{x}_1 x_2]_0^T - \int_0^T \bar{x}_1 u_2\, dt - \varepsilon \int_0^T \bar{x}_1 (f_2 u_1 + g_2 u_2)\, dt \\ &\quad + \varepsilon \int_0^T (f_3 u_1 + g_3 u_2)\, dt \\ &= [\bar{x}_1 x_2]_0^T - [\bar{x}_1 \bar{x}_2]_0^T + \int_0^T u_1 \bar{x}_2\, dt \\ &\quad - \varepsilon \int_0^T \bar{x}_1 (f_2 u_1 + g_2 u_2)\, dt + \varepsilon \int_0^T (f_3 u_1 + g_3 u_2)\, dt \\ &= -x_{30} - \varepsilon \int_0^T \bar{x}_1 (f_2 u_1 + g_2 u_2)\, dt + \varepsilon \int_0^T (f_3 u_1 + g_3 u_2)\, dt. \end{aligned}$$

Thus

$$x_3(T) = \varepsilon \int_0^T (\bar{x}_1 f_2 + f_3) u_1 + (\bar{x}_1 g_2 + g_3) u_2)\ dt. \tag{5}$$

For the fourth equation, similar computations yield

$$\begin{aligned} x_4(T) &= \varepsilon \int_0^T \left(\tfrac{\bar{x}_1^2}{2} f_2 - \bar{x}_1 f_3 + f_4 \right) u_1\ dt \\ &\quad + \varepsilon \int_0^T \left(\tfrac{\bar{x}_1^2}{2} g_2 - \bar{x}_1 g_3 + g_4 \right) u_2\ dt. \end{aligned} \tag{6}$$

Relations (4,5,6) lead to the following kinds of integrals ($i = 1, 2$):

$$\int_0^T f(x) u_i\, dt, \quad \int_0^T \bar{x}_1 f(x) u_i\, dt, \quad \int_0^T (\bar{x}_1)^2 f(x) u_i\, dt$$

where f is a smooth function. The essential part of the proof consists now to estimate these integrals. In fact, we have the following general estimation.

Lemma *For any smooth function $\mathbb{R}^4 \times \mathbb{R} \ni (x, \varepsilon) \mapsto f(x, \varepsilon) \in \mathbb{R}$ and $(r_1, r_2, r_3, r_4) \in \mathbb{N}^4$, there exist $\mu > 0$ and $D > 0$ such that, if $\|x^0\| \leq D$ and $|\varepsilon| \leq \mu$, then,*

$$\left| \int_0^T (\bar{x}_1)^{r_1} (\bar{x}_2)^{r_2} (\bar{x}_3)^{r_3} (\bar{x}_4)^{r_4} f(x(t), \varepsilon) u_i(t)\, dt \right| \leq D \|x^0\|,$$

for $i = 1, 2$. .

Proof of the lemma Set $a_1 = \min(\alpha_3, \alpha_4)$ and $a_2 = \min(1 - \alpha_3, 1 - 2\alpha_4)$. By assumption, $0 < a_1, a_2 < 1$, $a_1 + 2a_2 \geq 1$ and $3a_1 + a_2 \geq 1$. We use here Landau notation: a function $I(x^0)$ is $0(\|x^0\|^\beta)$, if exists a constant K independent of x^0 such that for x^0 close to 0, $\|I(x^0)\| \leq K \|x^0\|^\beta$.

Since $u_1 = 0(\|x^0\|^{a_1})$ and $u_2 = 0(\|x^0\|^{a_2})$, we have $\bar{x}_1 = 0(\|x^0\|^{a_1})$, $\bar{x}_2 = 0(\|x^0\|^{a_2})$, $\bar{x}_3 = 0(\|x^0\|^{a_1 + a_2})$, $\bar{x}_4 = 0(\|x^0\|^{2a_1 + a_2})$ and

$$(\bar{x}_1)^{r_1} (\bar{x}_2)^{r_2} (\bar{x}_3)^{r_3} (\bar{x}_4)^{r_4} = 0(\|x^0\|^{(r_1 + r_3 + 2r_4) a_1 + (r_2 + r_3 + r_4) a_2}).$$

Thus the lemma estimation is not obvious for only special values of the r_i's. Set n_1 and n_2 the largest integers such that $(n_1 + 1) a_1 < 1$ and $(n_2 + 1) a_2 < 1$. We have to consider the following finite cases corresponding to the following terms:

case 1: $u_1, \quad \bar{x}_1 u_1, \quad \ldots, \quad (\bar{x}_1)^{n_1} u_1$.
case 2: $u_1 \bar{x}_2, \quad u_1 \bar{x}_1 \bar{x}_2, \quad u_1 \bar{x}_3$.
case 3: $u_2, \bar{x}_2 u_2, \quad \ldots, \quad (\bar{x}_2)^{n_2} u_2$.
case 4: $\bar{x}_1 u_2, \quad (\bar{x}_1)^2 u_2$.

Take the integral $\int_0^T f u_1\ dt$. Using $u_1 = \dot{\bar{x}}_1$, an integration by part yields

$$\int_0^T f\, u_1\ dt = -x_1^0 f(x^0, \varepsilon) - \int_0^T \bar{x}_1 \left(\sum_{i=1}^4 \frac{\partial f}{\partial x_i} \dot{x}_i \right)\ dt.$$

The first term is $O(\|x^0\|)$. Its contribution is of correct order. We just have to deal with the second term. Substituting the $\dot{x}_i$'s by their expressions obtained from Σ_ε yields terms of the form $\int_0^T \bar{x}_1 h(x, \varepsilon) u_i\ dt$, with $i = 1, 2$ and $h(x, \varepsilon)$ smooth function combination of first derivatives of f with functions appearing in Σ_ε. Since $2x_1 u_1 = d/dt(\bar{x}_1)^2$ and $\bar{x}_1 u_2 = d/dt(\bar{x}_1 \bar{x}_2 - \bar{x}_3)$, another integration by part leads to $O(\|x^0\|)$ boundary terms and rest integrals of the form $\int_0^T (\bar{x}_1)^2 k(x, \varepsilon) u_i\ dt$ and $\int_0^T (\bar{x}_1 \bar{x}_2 - \bar{x}_3) l(x, \varepsilon) u_i\ dt$, with new smooth functions k and l.

The successive terms, involving $\bar{x}$ and u and generated by such calculations, are displayed on figure 1. The terms with a black dot are good terms, i.e. $0(\|x^0\|)$ terms. They do not belong to the previous list (case 1 to case 4). The three graphs of figure 1 can be be used as follows. Take, e.g., the bad term u_1: we have seen here above that one integration by part leads to $\bar{x}_1 u_1$ and $\bar{x}_1 u_2$. This is represented here by two arcs starting from u_1 and descending to $\bar{x}_1 u_1$ and $\bar{x}_1 u_2$. We see from this figure, that integrals with terms of case 2 or case 4 are, after few integrations by part, $O(\|x^0\|)$. Just integrals involving terms of

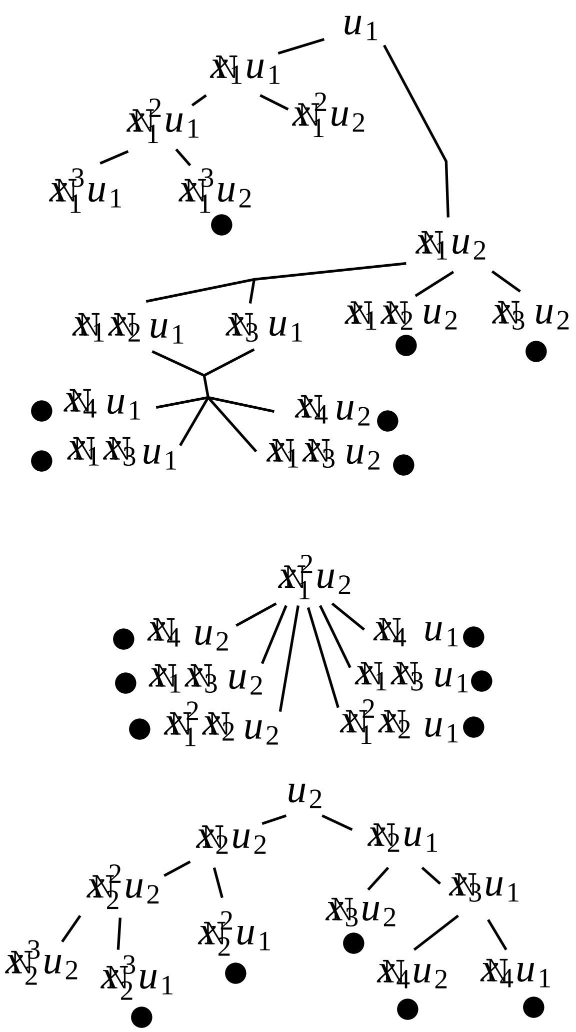

Figure 1: the terms obtained after successive integrations by part; a descending line means "an integration gives".

type $(\bar{x}_i)^k u_i$ remain to be estimated. We have

$$\int_0^T (\bar{x}_i)^k u_i h(x,\varepsilon)\,dt = \int_0^T (\bar{x}_i)^{k+1} u_i l(x,\varepsilon)\,dt + O(\|x^0\|).$$

where the smooth function l involves derivatives of h and the equations of Σ_ε. Since $\int_0^T (\bar{x}_i)^{n_i+1} u_i h(x,\varepsilon)\,dt = O(\|x^0\|)$, the integral with terms belonging to set 1 and set 2 satisfy also the lemma estimation. The lemma and theorem are thus proved.

Proposition *Consider $T > 0$, the chained system (1) with $n = 4$ and the initial condition $x^0 \in \mathbb{R}^4$. For $\alpha_3 \in]0,1[$ and $\alpha_4 \in]0,1/2[$, set*

$$\Delta = |x_1^0| + |x_2^0| + |x_3^0|^{\alpha_3} + |x_4^0|^{\alpha_4}.$$

Define the steering control $\mathcal{U} = (\mathcal{U}_1, \mathcal{U}_2)$ in two steps.

For $t \in [0, \frac{T}{2}]$**,** *set*

$$\mathcal{U}_1(x^0,t) = \frac{2(x_1^0 + \Delta)s'}{T}, \quad \mathcal{U}_2(x^0,t) = 0$$

with

$$\tau = \frac{2t}{T}, \quad s = 2\tau^3 - 3\tau^2, \quad s' = 6\tau(\tau - 1).$$

For $t \in [\frac{T}{2}, T]$ *: set*

$$\mathcal{U}_1(x_0,t) = \frac{2\Delta s'}{T}, \quad \mathcal{U}_2(x_0,t) = \frac{-2s'(60as^2 + 24bs + 6c)}{T}$$

with

$$\begin{aligned}
&\tau = \tfrac{2t}{T} - 1, \quad s = 3\tau^2 - 2\tau^3 - 1, \quad s' = 6\tau(1-\tau)\\
&\xi_3 = x_3^0 - x_2^0(x_1^0 + \Delta)\\
&\xi_4 = x_4^0 - x_3^0(x_1^0 + \Delta) + \tfrac{x_2^0}{2}(x_1^0 + \Delta)^2\\
&a = 6\tfrac{\xi_4}{\Delta^2} + 3\tfrac{\xi_3}{\Delta} + \tfrac{x_2^0}{2}, \quad b = 15\tfrac{\xi_4}{\Delta^2} + 7\tfrac{\xi_3}{\Delta} + x_2^0\\
&c = 10\tfrac{\xi_4}{\Delta^2} + 4\tfrac{\xi_3}{\Delta} + \tfrac{x_2^0}{2}.
\end{aligned}$$

Then, the open-loop control $\mathcal{U}$ steers (1) from x^0 at $t = 0$ to 0 at $t = T$. The dependence of $\mathcal{U}$ with respect to t is smooth with $\mathcal{U}(x^0,0) = \mathcal{U}(T,x^0) = 0$, for all x^0. The dependence of $\mathcal{U}$ with respect to x^0 is smooth excepted in 0 where it is continuous with $\mathcal{U}(0,t) = 0$,

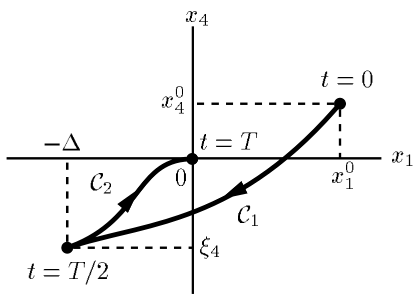

Figure 2: A piecewise polynomial steering trajectory of the flat output (x_1, x_4) for a chained system of dimension 4.

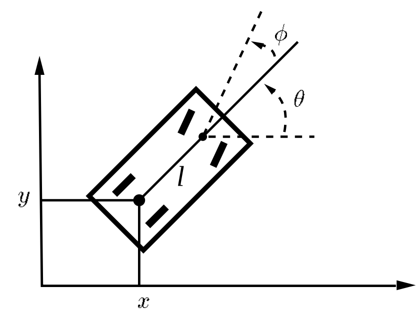

Figure 3: the car and the notation

for all $t \in [0, T]$. Moreover there exist k_1, k_2, two constants independent of $x^0 \in \mathbb{R}^4$ and $t \in [0, T]$ such that estimation (2) are satisfied.

The construction of this open-loop control relies on the general motion planning method explained in [12, 13, 8] and valid for flat systems. In the (x_1, x_4) plane, the flat output space, the curve $[0, T] \ni t \mapsto (x_1, x_4)$ generated by this control admits two smooth parts, $\mathcal{C}_1$ and $\mathcal{C}_2$ (see figure 2). $\mathcal{C}_1$, corresponding to $t \in [0, T/2]$, is a polynomial of degree 2, Π_1:

$$x_4 = \Pi_1(x_1) = x_4^0 + x_3^0(x_1 - x_1^0) + \frac{x_2^0}{2}(x_1 - x_1^0)^2.$$

$\mathcal{C}_2$, corresponding to $t \in [T/2, T]$, is the unique polynomial $x_4 = \Pi_2(x_1)$ of degree 5 such that

$$\frac{d^\nu \Pi_2}{dx_1^\nu}(-\Delta) = \frac{d^\nu \Pi_1}{dx_1^\nu}(-\Delta), \quad \frac{d^\nu \Pi_2}{dx_1^\nu}(0) = 0, \quad \nu = 0, 1, 2.$$

This simple geometric construction underlies the open-loop control described in the previous proposition. Notice that the "cusp" at $t = T/2$ is important to guaranty the regularity with respect to x^0 around 0 and the continuity at 0. The detailed proof of the proposition is straightforward and left to the reader.

3 Car-like robot

Consider the car-like robot of figure 3 considered for the first time in [9]. The equations are as follows.

$$\dot{x} = \cos\theta\ u, \quad \dot{y} = \sin\theta\ u, \quad \dot{\theta} = \tan\varphi\ \frac{u}{l}, \quad \dot{\varphi} = v$$

with two controls (u, v). Around 0, this system is flat ((x, y) is the flat output) and feedback equivalent to the chained system (1) with $n = 4$ via the following change of coordinates

$$x_1 = x, \quad x_2 = \frac{\tan\varphi}{l\cos^3\theta}, \quad x_3 = \tan\theta, \quad x_4 = y. \tag{7}$$

and static feedback,

$$u_1 = \cos\theta\ u, \quad u_2 = \frac{v}{l\cos^3\theta\cos^2\varphi} + \frac{3\sin\theta\tan^2\varphi\ u}{l^2\cos^4\theta}. \tag{8}$$

For the simulation of figure 4, we have $l = 1.8\ m$. The feedback (8) is first used and (u_1, u_2) is computed according to the previous proposition with $\alpha_3 = \alpha_4 = 1/4$ and $T = 1$. To check the control robustness ensured by the theorem, we introduce in the simulations the following errors: for the control l is underestimate of 20%, i.e. $l = 1.5\ m$, and the car velocity u is overestimated of 15%. Figure 4, shows that, in spite of these rather large systematic errors, the convergence to 0 is achieved in practice after 4 iterations, i.e. $t > 4T = 4$.

4 Conclusion

As demonstrated here above, the possibility of robust stabilization for non flat systems Σ_ε through the use of flat approximations Σ_0 prolongs a well known and widely use method. This method consists in stabilizing a nonlinear systems around equilibria via their

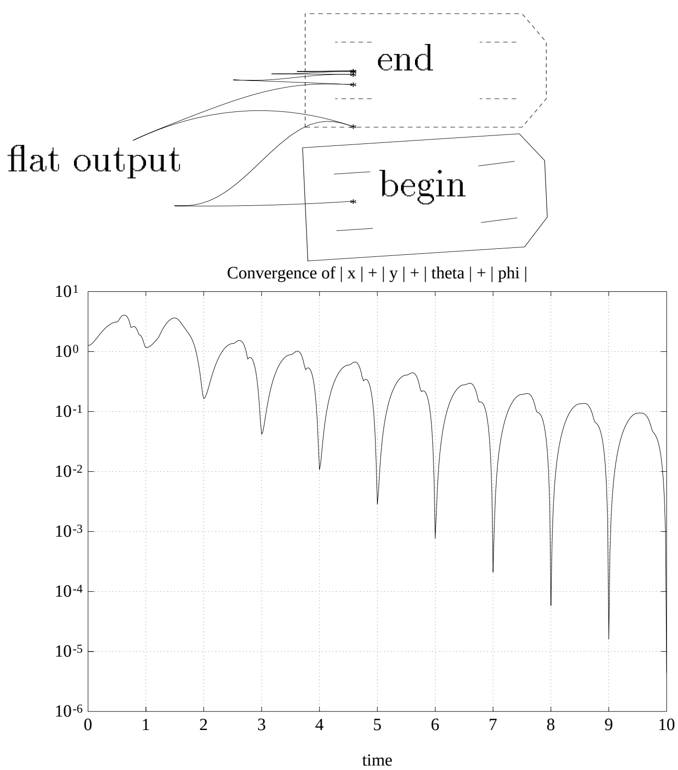

Figure 4: robust stabilization of the car; error of +20% for the length l and -15% for the velocity u.

first order tangent approximations when it is controllable (i.e., flat). This paper indicates that extending linear controllable (or flat) approximations to nonlinear flat approximations can be interesting.

References

[1] M.K. Bennani. Commande non linéaire de véhicules sur roues avec remorques. Technical report, École Polytechnique, Paris, option de fin d'étude, July 1994.

[2] A.M. Bloch, N.H. McClamroch, and M. Reyhanoglu. Control and stabilization of nonholonomic dynamic systems. *IEEE Trans. Automat. Control*, 37:1746–1757, 1992.

[3] C Canudas de Wit, H. Berghuis, and H. Nijmeijer. Hybrid stabilization of nonlinear systems in chained form. *preprint and submitted for publication*, 1993.

[4] C Canudas de Wit and O.J. Sørdalen. Exponential stabilization of mobile robots with nonholonomic constraints. *IEEE Trans. Automat. Control*, 37:1791–1797, 1992.

[5] J.M. Coron. Global stabilization for controllable systems without drift. *Math. Control Signals Systems*, 5:295–312, 1992.

[6] J.M. Coron. Linearized control systems and applications to smooth stabilization. *SIAM J. Control Optimization*, 32:358–386, 1994.

[7] J.M. Coron. On the stabilization in finite time of locally controllable systems by means of continuous time-varying feedback laws. *Preprint Univertité Paris-Sud*, (9228), 1994.

[8] M. Fliess, J. Lévine, Ph. Martin, and P. Rouchon. Flatness and defect of nonlinear systems: introductory theory and examples. *Internat. J. Control*, 1995.

[9] J.P. Laumond and T. Siméon. Motion planning for a two degrees of freedom mobile with towing. In *IEEE International Conf. on Control and Applications*, 1989.

[10] R.M. Murray and S.S. Sastry. Nonholonomic motion planning: Steering using sinusoids. *IEEE Trans. Automat. Control*, 38:700–716, 1993.

[11] J.B. Pomet. Explicit design of time-varying stabilizing control laws for a class of controllable systems without drift. *Systems Control Letters*, 18:147–158, 1992.

[12] P. Rouchon, M. Fliess, J. Lévine, and Ph. Martin. Flatness and motion planning: the car with n-trailers. In *Proc. ECC'93, Groningen*, pages 1518–1522, 1993.

[13] P. Rouchon, M. Fliess, J. Lévine, and Ph. Martin. Flatness, motion planning and trailer systems. In *Proc. 32nd IEEE Conf. Decision and Control, San Antonio*, pages 2700–2705, december 1993.

[14] C. Samson. Time-varying feedback stabilization of carlike wheeled mobile robots. *Internat. J. Robotics Res.*, 12:55–64, 1993.

[15] C. Samson. Control of chained systems. application to path following and time-varying point-stabilization of mobile robots. *IEEE Trans. Automat. Control*, 40:64–77, 1995.

[16] O.J. Sørdalen and O. Egeland. Exponential stabilization of chained nonholonomic systems. In *Proc. ECC'93, Groningen*, pages 1438–1443, 1993.

[17] O.J. Sørdalen and O. Egeland. Exponential stabilization of nonholonomic chained systems. *IEEE Trans. Automat. Control*, 40:35–49, 1995.

[18] D. Tilbury and A. Chelouah. Steering a three-input nonholonomic using multirate controls. In *Proc. ECC'93, Groningen*, pages 1428–1431, 1993.

[19] G. Walsh, D. Tilbury, S. Sastry, R. Murray, and J.P. Laumond. Stabilization of trajectories for systems with nonholonomic constraints. *IEEE Trans. Automat. Control*, 39:216–222, 1994.