On the first experimental realization of a quantum state feedback

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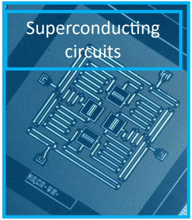
Technologies for quantum simulation and computation[1]



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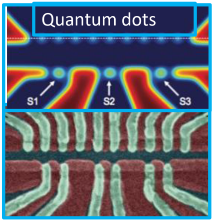

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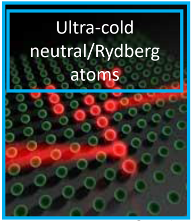

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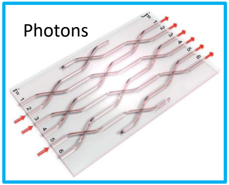

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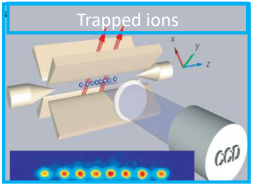

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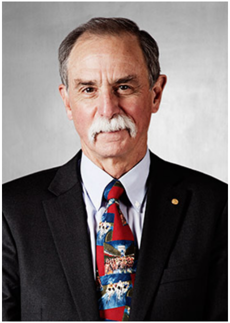
Requirement:

- Scalable modular architecture
- Control software from the very beginning

[1] Courtesy of Walter Riess, IBM Research - Zurich.

Nobel Prize in Physics 2012

Serge Haroche



David J. Wineland

" This year's Nobel Prize in Physics honours the experimental inventions and discoveries that have allowed the **measurement and control of individual quantum systems***. They belong to two separate but related technologies: ions in a harmonic trap and photons in a cavity"*

From the Scientific Background on the Nobel Prize in Physics 2012 compiled by the Class for Physics of the Royal Swedish Academy of Sciences, October 9, 2012.

Outline

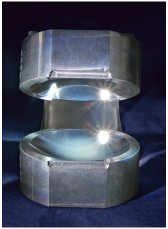
The photon-box experiment of the Haroche group
- Modelling based on three quantum rules
- Measuring photons without destroying them
- Stabilizing measurement-based feedback

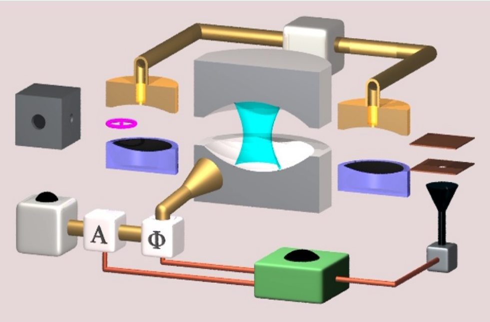
Model structures (decoherence, measurement back-action)
- Discrete-time models (hidden Markov chain)
- Diffusive models (Wiener process)
- Jump models (Poisson process)

Feedback for quantum systems
- Measurement-based feedback
- Coherent (autonomous) feedback and dissipation engineering

The first experimental realization of a quantum-state feedback

microwave photons
(10 GHz)





Experiment: C. Sayrin, I. Dotsenko, X. Zhou, B. Peaudecerf, T. Rybarczyk, S. Gleyzes, P. Rouchon, M. Mirrahimi, H. Amini, M. Brune, J.M. Raimond, **S. Haroche**: Real-time quantum feedback prepares and stabilizes photon number states. Nature, **2011**, 477, 73-77.

Theory: I. Dotsenko, M. Mirrahimi, M. Brune, **S. Haroche**, J.M. Raimond, P. Rouchon: Quantum feedback by discrete quantum non-demolition measurements: towards on-demand generation of photon-number states. Physical Review A, **2009**, 80: 013805-013813.
M. Mirrahimi et al. CDC 2009, 1451-1456, **2009**.
H. Amini et al. IEEE Trans. Automatic Control, 57 (8): 1918–1930, **2012**.
R. Somaraju et al., Rev. Math. Phys., 25, 1350001, **2013**.
H. Amini et. al., Automatica, 49 (9): 2683-2692, **2013**.

Three quantum features emphasized by the photon box[2]

1. Schrödinger: wave funct. $|\psi\rangle$ of norm one in Hilbert space $\mathcal{H}$

$$\frac{d}{dt}|\psi\rangle = -\frac{i}{\hbar}\boldsymbol{H}|\psi\rangle, \quad \boldsymbol{H} = \boldsymbol{H}_0 + u\boldsymbol{H}_1, \quad \boldsymbol{u \text{ classical control input}}$$

Dirac notations: $|\psi\rangle \equiv \begin{pmatrix} \bullet \\ \bullet \end{pmatrix}$, $|\psi\rangle^\dagger = \langle\psi| \equiv \begin{pmatrix} \bullet & \bullet \end{pmatrix}$, $\boldsymbol{H} = \boldsymbol{H}^\dagger \equiv \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$.

2. Origin of dissipation: collapse of the wave packet induced by the measurement of observable $\boldsymbol{O}$ with spectral decomp. $\sum_y \lambda_y \boldsymbol{P}_y$:
   - **measured output y** with probability $\mathbb{P}_y = \langle\psi|\boldsymbol{P}_y|\psi\rangle$;
   - **back-action**:

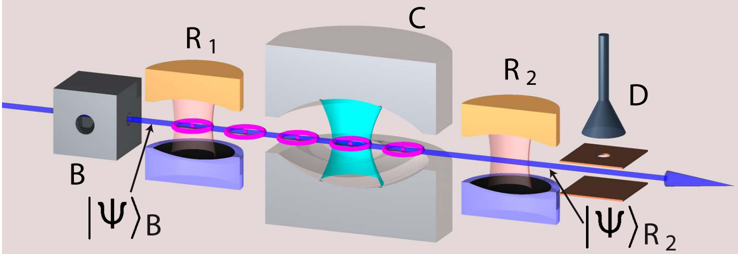
$$|\psi\rangle \mapsto |\psi\rangle_+ = \frac{\boldsymbol{P}_y|\psi\rangle}{\sqrt{\langle\psi|\boldsymbol{P}_y|\psi\rangle}}$$

3. Tensor product for composite system (S, M):
   - Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_M$
   - Hamiltonian $\boldsymbol{H} = \boldsymbol{H}_S \otimes \boldsymbol{I}_M + \boldsymbol{H}_{int} + \boldsymbol{I}_S \otimes \boldsymbol{H}_M$
   - observable on sub-system M only: $\boldsymbol{O} = \boldsymbol{I}_S \otimes \boldsymbol{O}_M$.

[2] S. Haroche and J.M. Raimond. Exploring the Quantum: Atoms, Cavities and Photons. Oxford Graduate Texts, 2006.

Composite system (S, M): harmonic oscillator $\otimes$ qubit.

- System S corresponds to a quantized harmonic oscillator:
$$\mathcal{H}_S = \left\{ \sum_{n=0}^{\infty} x_n \left|n\right\rangle \,\middle|\, (x_n)_{n=0}^{\infty} \in l^2(\mathbb{C}) \right\},$$
where $|n\rangle$ is the photon-number state with n photons ($\langle n_1|n_2\rangle = \delta_{n_1,n_2}$).
- Meter M is a qubit, a 2-level system:
$$\mathcal{H}_M = \left\{ x_g \left|g\right\rangle + x_e|e\rangle \,\middle|\, x_g, x_e \in \mathbb{C} \right\},$$
where $|g\rangle$ (resp. $|e\rangle$) is the ground (resp. excited) state ($\langle g|g\rangle = \langle e|e\rangle = 1$ and $\langle g|e\rangle = 0$)
- **State of the composite system $|\Psi\rangle \in \mathcal{H}_S \otimes \mathcal{H}_M$:**
$$|\Psi\rangle = \sum_{n=0}^{+\infty} x_{ng} \left|n\right\rangle \otimes \left|g\right\rangle + x_{ne} \left|n\right\rangle \otimes \left|e\right\rangle, \qquad x_{ne}, x_{ng} \in \mathbb{C}.$$
Ortho-normal basis: $\big(|n\rangle \otimes |g\rangle, |n\rangle \otimes |e\rangle\big)_{n\in\mathbb{N}}$.



- When atom comes out B, the quantum state $|\boldsymbol{\Psi}\rangle_B$ of the composite system is separable: $|\boldsymbol{\Psi}\rangle_B = |\psi\rangle \otimes |g\rangle$.
- Just before the measurement in D, the state is in general entangled (not separable):

$$|\boldsymbol{\Psi}\rangle_{R_2} = \boldsymbol{U}_{SM}\big(|\psi\rangle \otimes |g\rangle\big) = \big(\boldsymbol{M}_g|\psi\rangle\big) \otimes |g\rangle + \big(\boldsymbol{M}_e|\psi\rangle\big) \otimes |e\rangle$$

where $\boldsymbol{U}_{SM}$ is a unitary transformation (Schrödinger propagator) defining measurement operators $\boldsymbol{M}_g$ and $\boldsymbol{M}_e$ on $\mathcal{H}_S$.
$\boldsymbol{U}_{SM}$ unitary implies $\boldsymbol{M}_g^\dagger \boldsymbol{M}_g + \boldsymbol{M}_e^\dagger \boldsymbol{M}_e \equiv \boldsymbol{I}$.

Just before the atom detector D the quantum state is **entangled**:

$$|\boldsymbol{\Psi}\rangle_{R_2} = \boldsymbol{M}_g|\psi\rangle \otimes |g\rangle + \boldsymbol{M}_e|\psi\rangle \otimes |e\rangle$$

Just after outcome[3] $\boldsymbol{y} \in \{\boldsymbol{g}, \boldsymbol{e}\}$, the state becomes **separable**:

$$|\boldsymbol{\Psi}\rangle_D = \left(\frac{\boldsymbol{M}_y}{\sqrt{\langle\psi|\boldsymbol{M}_y^\dagger \boldsymbol{M}_y|\psi\rangle}}|\psi\rangle\right) \otimes |y\rangle \quad \text{with probability } \langle\psi|\boldsymbol{M}_y^\dagger \boldsymbol{M}_y|\psi\rangle.$$

Hidden Markov chain: $|\psi\rangle \otimes |g\rangle \longrightarrow \left(\frac{\boldsymbol{M}_y}{\sqrt{\langle\psi|\boldsymbol{M}_y^\dagger \boldsymbol{M}_y|\psi\rangle}}|\psi\rangle\right) \otimes |y\rangle$

$$|\psi_{k+1}\rangle = \begin{cases} \frac{\boldsymbol{M}_g}{\sqrt{\langle\psi_k|\boldsymbol{M}_g^\dagger \boldsymbol{M}_g|\psi_k\rangle}}|\psi_k\rangle, & y_k = g \text{ with probability } \langle\psi_k|\boldsymbol{M}_g^\dagger \boldsymbol{M}_g|\psi_k\rangle; \\ \frac{\boldsymbol{M}_e}{\sqrt{\langle\psi_k|\boldsymbol{M}_e^\dagger \boldsymbol{M}_e|\psi_k\rangle}}|\psi_k\rangle, & y_k = e \text{ with probability } \langle\psi_k|\boldsymbol{M}_e^\dagger \boldsymbol{M}_e|\psi_k\rangle; \end{cases}$$

with state $|\psi_k\rangle$ and output $y_k \in \{g, e\}$ at time-step k:

[3] Measurement operator $\boldsymbol{O} = \boldsymbol{I}_S \otimes (|e\rangle\langle e| - |g\rangle\langle g|)$.

Assume known $|\psi_0\rangle$ and **detector out of order**: **what about** $|\psi_1\rangle$ **?**

- Expectation value of $|\psi_1\rangle\langle\psi_1|$ knowing $|\psi_0\rangle$:[4]

$$\mathbb{E}\Big(\underbrace{|\psi_1\rangle\langle\psi_1|}_{\rho_1}\ \Big|\ |\psi_0\rangle\Big) = \boldsymbol{M}_g \underbrace{|\psi_0\rangle\langle\psi_0|}_{\rho_0} \boldsymbol{M}_g^\dagger + \boldsymbol{M}_e \underbrace{|\psi_0\rangle\langle\psi_0|}_{\rho_0} \boldsymbol{M}_e^\dagger.$$

- Set $\boldsymbol{K}(\rho) \triangleq \boldsymbol{M}_g \rho \boldsymbol{M}_g^\dagger + \boldsymbol{M}_e \rho \boldsymbol{M}_e^\dagger$ for any operator ρ.
- ρ_k expectation of $|\psi_k\rangle\langle\psi_k|$ knowing $\rho_0 = |\psi_0\rangle\langle\psi_0|$:

$$\rho_1 = \boldsymbol{K}(\rho_0), \quad \rho_2 = \boldsymbol{K}(\rho_1), \quad \ldots \quad, \rho_k = \boldsymbol{K}(\rho_{k-1}).$$

Linear map $\boldsymbol{K}$: positivity and trace preserving ($\boldsymbol{M}_g^\dagger \boldsymbol{M}_g + \boldsymbol{M}_e^\dagger \boldsymbol{M}_e = \boldsymbol{I}$) called Kraus map or quantum channel.

Density operators ρ: Hermitian non-negative operators of trace one.

[4] $|\psi\rangle\langle\psi|$: orthogonal projector on line spanned by unitary vector $|\psi\rangle$.

The Markov chain with ρ as hidden state

Detector efficiency $\eta \in [0, 1]$. Measured output $y \in \{g, e, \varnothing\}$:

$$\rho_{k+1} = \begin{cases} \frac{\boldsymbol{K}_g(\rho_k)}{\mathrm{Tr}\left(\boldsymbol{K}_g(\rho_k)\right)}, \ y_k = g \text{ with probability } \mathrm{Tr}\left(\boldsymbol{K}_g(\rho_k)\right); \\ \frac{\boldsymbol{K}_e(\rho_k)}{\mathrm{Tr}\left(\boldsymbol{K}_e(\rho_k)\right)}, \ y_k = e \text{ with probability } \mathrm{Tr}\left(\boldsymbol{K}_e(\rho_k)\right); \\ \frac{\boldsymbol{K}_\varnothing(\rho_k)}{\mathrm{Tr}\left(\boldsymbol{K}_\varnothing(\rho_k)\right)}, \ y_k = \varnothing \text{ with probability } \mathrm{Tr}\left(\boldsymbol{K}_\varnothing(\rho_k)\right); \end{cases}$$

with Kraus maps

$$\boldsymbol{K}_g(\rho) = \eta \boldsymbol{M}_g \rho \boldsymbol{M}_g^\dagger, \qquad \boldsymbol{K}_e(\rho) = \eta \boldsymbol{M}_e \rho \boldsymbol{M}_e^\dagger$$

$$\boldsymbol{K}_\varnothing(\rho) = (1-\eta)\left(\boldsymbol{M}_g \rho \boldsymbol{M}_g^\dagger + \boldsymbol{M}_e \rho \boldsymbol{M}_e^\dagger\right).$$

We still have:

$$\mathbb{E}\left(\rho_{k+1} \mid \rho_k\right) \triangleq \boldsymbol{K}(\rho_k) = \boldsymbol{M}_g \rho_k \boldsymbol{M}_g^\dagger + \boldsymbol{M}_e \rho_k \boldsymbol{M}_e^\dagger = \sum_y \boldsymbol{K}_y(\rho_k).$$

Imperfections, decoherence modeled similarly

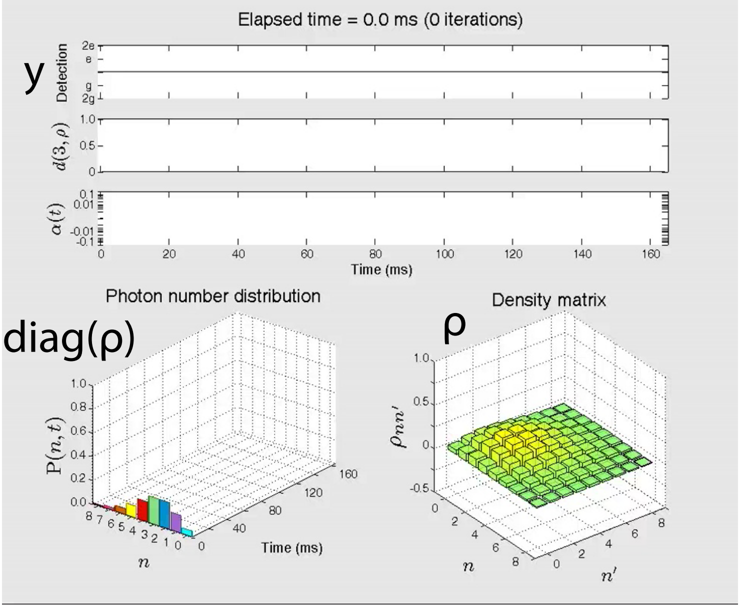
A controlled Markov process (input u, hidden state ρ, output y)

Input u: classical amplitude of a coherent micro-wave pulse
State ρ: the density operator of the photon(s) trapped in the cavity
Output y: measurement of the probe atom

$$\rho_{k+1} = \begin{cases} \dfrac{\boldsymbol{D}_{u_k}\boldsymbol{K}_g(\rho_k)\boldsymbol{D}_{u_k}^\dagger}{\text{Tr}\left(\boldsymbol{K}_g(\rho_k)\right)}, y_k = g \text{ with probability } \text{Tr}\left(\boldsymbol{K}_g(\rho_k)\right); \\ \dfrac{\boldsymbol{D}_{u_k}\boldsymbol{K}_e(\rho_k)\boldsymbol{D}_{u_k}^\dagger}{\text{Tr}\left(\boldsymbol{K}_e(\rho_k)\right)}, y_k = e \text{ with probability } \text{Tr}\left(\boldsymbol{K}_e(\rho_k)\right); \\ \dfrac{\boldsymbol{D}_{u_k}\boldsymbol{K}_\varnothing(\rho_k)\boldsymbol{D}_{u_k}^\dagger}{\text{Tr}\left(\boldsymbol{K}_\varnothing(\rho_k)\right)}, y_k = \varnothing \text{ with probability } \text{Tr}\left(\boldsymbol{K}_\varnothing(\rho_k)\right); \end{cases}$$

Controlled displacement unitary operator ($u \in \mathbb{R}$): $\boldsymbol{D}_u = e^{\boldsymbol{u}\,(\boldsymbol{a}^\dagger - \boldsymbol{a})}$ with $\boldsymbol{a} = $ upper diag$(\sqrt{1}, \sqrt{2}, \ldots)$ the photon annihilation operator.
Measurement Kraus operators with linear dispersive interaction
$\boldsymbol{M}_g = \cos\left(\frac{\phi_0 \boldsymbol{N} + \phi_R}{2}\right)$ and $\boldsymbol{M}_e = \sin\left(\frac{\phi_0 \boldsymbol{N} + \phi_R}{2}\right)$: $\boldsymbol{M}_g^\dagger \boldsymbol{M}_g + \boldsymbol{M}_e^\dagger \boldsymbol{M}_e = \boldsymbol{I}$ with $\boldsymbol{N} = \boldsymbol{a}^\dagger \boldsymbol{a} = $ diag$(0, 1, 2, \ldots)$ the photon number operator.

Open-loop dynamics ($u = 0$): experimental data



$$\rho_{k+1} = \begin{cases} \frac{\boldsymbol{K}_g(\rho_k)}{\text{Tr}(\boldsymbol{K}_g(\rho_k))}, y_k = g \text{ with probability } \text{Tr}\left(\boldsymbol{K}_g(\rho_k)\right); \\ \frac{\boldsymbol{K}_e(\rho_k)}{\text{Tr}(\boldsymbol{K}_e(\rho_k))}, y_k = e \text{ with probability } \text{Tr}\left(\boldsymbol{K}_e(\rho_k)\right); \\ \frac{\boldsymbol{K}_\varnothing(\rho_k)}{\text{Tr}(\boldsymbol{K}_\varnothing(\rho_k))}, y_k = \varnothing \text{ with probability } \text{Tr}\left(\boldsymbol{K}_\varnothing(\rho_k)\right); \end{cases}$$

Photon-number state $|\bar{\boldsymbol{n}}\rangle\langle\bar{\boldsymbol{n}}|$ is a steady-state: $\boldsymbol{K}_y(|\bar{\boldsymbol{n}}\rangle\langle\bar{\boldsymbol{n}}|) \propto |\bar{\boldsymbol{n}}\rangle\langle\bar{\boldsymbol{n}}|$.

Martingales $W_g(\rho) = \text{Tr}\left(g(\boldsymbol{N})\rho\right)$ for any function g:

$$\mathbb{E}\left(W_g(\rho_{k+1}) \;/\; \rho_k\right) = W_g(\rho_k).$$

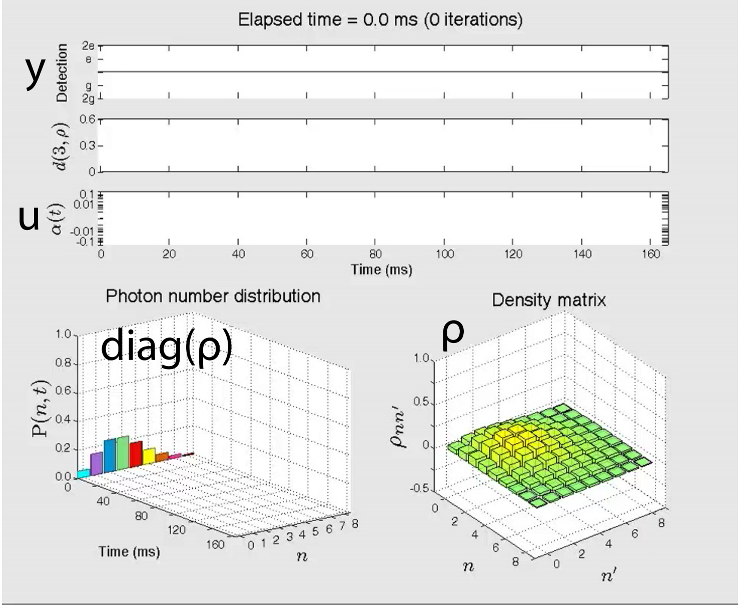
Convergence: $W(\rho) = 1 - \sum_n \langle n\,|\rho|\,n\rangle^2$ super-martingale,

$$\mathbb{E}\left(W(\rho_{k+1}) \;/\; \rho_k\right) = W(\rho_k) - Q(\rho_k)$$

where $Q(\rho) \geq 0$ and $Q(\rho) = 0$ iff $\exists \bar{\boldsymbol{n}}$ such that $\rho = |\bar{\boldsymbol{n}}\rangle\langle\bar{\boldsymbol{n}}|$.

Probability to converge to $|\bar{\boldsymbol{n}}\rangle\langle\bar{\boldsymbol{n}}|$: $\text{Tr}\left(|\bar{\boldsymbol{n}}\rangle\langle\bar{\boldsymbol{n}}|\rho_0\right) = \langle\bar{n}|\rho_0|\bar{n}\rangle$.

Feedback stabilization around 3-photon state: experimental data



Structure of the stabilizing quantum-state feedback scheme

With a sampling time of 80 μs, the controller is classical

- Goal: stabilization towards $|\bar{\boldsymbol{n}}\rangle\langle\bar{\boldsymbol{n}}|$.
- Step $k-1$ to step k
  1. read y_{k-1} the measurement outcome for probe atom $k-1$.
  2. compute $\boldsymbol{\rho_k}$ from $\boldsymbol{\rho_{k-1}}$ via $\rho_k = \dfrac{\boldsymbol{D}_{u_{k-1}} \boldsymbol{K}_{y_{k-1}}(\rho_{k-1}) \boldsymbol{D}^{\dagger}_{u_{k-1}}}{\mathrm{Tr}\left(\boldsymbol{K}_{y_{k-1}}(\rho_{k-1})\right)}$,
  3. compute u_k as a function of $\boldsymbol{\rho_k}$ (state feedback).
  4. apply the micro-wave pulse of amplitude u_k.

Observer/controller structure:

1. real-time state estimation: the quantum Belavkin filter
2. $u = f(\rho)$ based on a strict control Lyapunov function $V(\rho)$ derived from open-loop martingales $\mathrm{Tr}\left(g(\boldsymbol{N})\rho\right)$:

$$\mathbb{E}\left(V(\rho_{k+1}) \,\middle|\, \rho_k, u_k = f(\rho_k)\right) = V(\rho_k) - Q(\rho_k)$$

where $Q(\rho) \geq 0$ and $Q(\rho) = 0$ iff $\rho = |\bar{\boldsymbol{n}}\rangle\langle\bar{\boldsymbol{n}}|$.

Experimental closed-loop data

C. Sayrin et. al., Nature 477, 73-77, Sept. 2011.

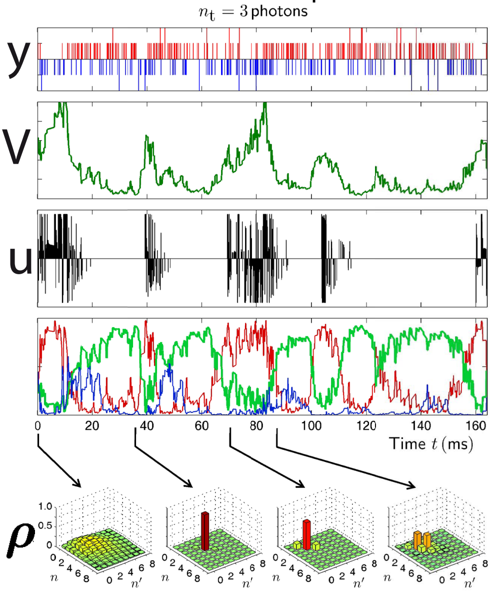
Decoherence due to finite photon life-time (70 ms)

Detection efficiency 40%
Detection error rate 10%
Delay 4 sampling periods

The quantum filter includes decoherence, detector imperfections and delays (Bayes law).

Truncation to 9 photons

Stabilization around 3-photon state



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Feedback for quantum systems
- Measurement-based feedback
- Coherent (autonomous) feedback and dissipation engineering

Discrete-time Stochastic Master Equations (SME)

Trace preserving Kraus map $\boldsymbol{K}_u$ depending on u:

$$\boldsymbol{K}_u(\boldsymbol{\rho}) = \sum_{\xi} \boldsymbol{M}_{u,\xi} \boldsymbol{\rho} \boldsymbol{M}_{u,\xi}^\dagger \quad \text{with} \quad \sum_{\xi} \boldsymbol{M}_{u,\xi}^\dagger \boldsymbol{M}_{u,\xi} = \boldsymbol{I}.$$

Take a left stochastic matrix $\left[\eta_{y,\xi}\right]$ and set $\boldsymbol{K}_{u,y}(\boldsymbol{\rho}) = \sum_{\xi} \eta_{y,\xi} \boldsymbol{M}_{u,\xi} \boldsymbol{\rho} \boldsymbol{M}_{u,\xi}^\dagger$.
Associated Markov chain reads:

$$\boldsymbol{\rho}_{k+1} = \frac{\boldsymbol{K}_{u_k,y_k}(\boldsymbol{\rho}_k)}{\operatorname{Tr}\left(\boldsymbol{K}_{u_k,y_k}(\boldsymbol{\rho}_k)\right)} \quad \text{with } y_k \text{ with probability } \operatorname{Tr}\left(\boldsymbol{K}_{u_k,y_k}(\boldsymbol{\rho}_k)\right).$$

Classical input u, hidden state $\boldsymbol{\rho}$, measured output y.
Ensemble average given by $\boldsymbol{K}_u$ since $\mathbb{E}\left(\boldsymbol{\rho}_{k+1} \mid \boldsymbol{\rho}_k, u_k\right) = \boldsymbol{K}_{u_k}(\boldsymbol{\rho}_k)$.
Markov model useful for:

1. Monte-Carlo simulations of quantum trajectories (decoherence, measurement back-action).
2. quantum filtering to get the quantum state $\boldsymbol{\rho}_k$ from $\boldsymbol{\rho}_0$ and $(y_0, \ldots, y_{k-1})$ (Belavkin quantum filter developed for diffusive models).
3. feedback design and Monte-Carlo closed-loop simulations.

- Lindblad equation ($\boldsymbol{H}$ Hermitian, $\boldsymbol{L}$ arbitrary, depending on u):

$$\frac{d}{dt}\rho \triangleq \mathcal{L}(\rho) = \underbrace{-\tfrac{i}{\hbar}[\boldsymbol{H},\rho]}_{\text{Schrödinger}} + \underbrace{\boldsymbol{L}\rho\boldsymbol{L}^\dagger - \frac{1}{2}(\boldsymbol{L}^\dagger\boldsymbol{L}\rho + \rho\boldsymbol{L}^\dagger\boldsymbol{L})}_{\triangleq \mathcal{D}_{\boldsymbol{L}}(\rho) \text{ decoherence}}$$

For any $t \geq 0$, $\rho_0 \mapsto e^{t\mathcal{L}}(\rho_0) = \rho_t$ is a trace preserving Kraus map.

- Continuous-time output $t \mapsto \boldsymbol{y_t}$ with $\boldsymbol{dy_t} = \sqrt{\eta}\,\text{Tr}\left((\boldsymbol{L}+\boldsymbol{L}^\dagger)\,\rho_t\right)\,dt + \boldsymbol{dW_t}$:

$$d\rho_t = \left(-\tfrac{i}{\hbar}[\boldsymbol{H},\rho_t] + \boldsymbol{L}\rho_t\boldsymbol{L}^\dagger - \frac{1}{2}(\boldsymbol{L}^\dagger\boldsymbol{L}\rho_t + \rho_t\boldsymbol{L}^\dagger\boldsymbol{L})\right)dt$$
$$+ \sqrt{\eta}\left(\boldsymbol{L}\rho_t + \rho_t\boldsymbol{L}^\dagger - \text{Tr}\left((\boldsymbol{L}+\boldsymbol{L}^\dagger)\rho_t\right)\rho_t\right)\boldsymbol{dW_t}$$

driven by Wiener process $\boldsymbol{dW_t}$ (η detection efficiency).

- Another formulation with Itō differentiation rule:

$$\rho_{t+dt} = \frac{\boldsymbol{K}_{\boldsymbol{dy_t}}(\rho)}{\text{Tr}\left(\boldsymbol{K}_{\boldsymbol{dy_t}}(\rho)\right)} \text{ with proba. density } \text{Tr}\left(\boldsymbol{K}_{\boldsymbol{dy_t}}(\rho)\right) \text{ for } \boldsymbol{dy_t}$$

$$\boldsymbol{K}_{\boldsymbol{dy}}(\rho) = e^{-\frac{\boldsymbol{dy^2}}{2dt}}\left(\boldsymbol{M}_{\boldsymbol{dy}}\rho\boldsymbol{M}_{\boldsymbol{dy}}^\dagger + (1-\eta)\boldsymbol{L}\rho_t\boldsymbol{L}^\dagger dt\right)$$
$$\boldsymbol{M}_{\boldsymbol{dy}} = \boldsymbol{I} + \left(-\tfrac{i}{\hbar}\boldsymbol{H} - \tfrac{1}{2}\left(\boldsymbol{L}^\dagger\boldsymbol{L}\right)\right)dt + \sqrt{\eta}\,\boldsymbol{dy}\,\boldsymbol{L}.$$

Stochastic master equation driven by Poisson process

- Lindblad equation: $\frac{d}{dt}\rho \triangleq \mathcal{L}(\rho) = -\frac{i}{\hbar}[H,\rho] + L\rho L^\dagger - \frac{1}{2}(L^\dagger L\rho + \rho L^\dagger L)$
- Counter $t \mapsto y(t) \in \mathbb{N}$ (detection imperfections $\vartheta \geq 0$, $\eta \in [0,1]$):

$$dy(t) = 0 \text{ or } 1 : \quad \mathbb{E}\left(dy(t) \mid \rho_t\right) = \left(\vartheta + \eta \operatorname{Tr}\left(L\rho_t L^\dagger\right)\right) dt$$

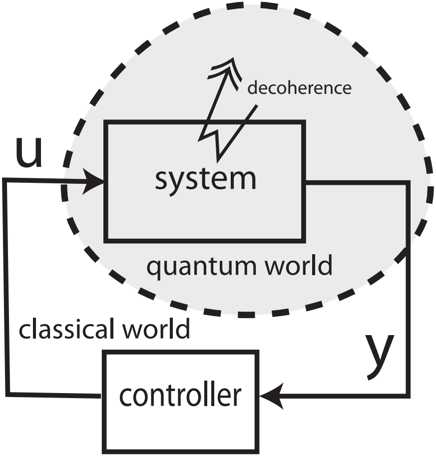
- Dynamics:

$$d\rho_t = \left(-\frac{i}{\hbar}[H,\rho_t] + L\rho_t L^\dagger - \frac{1}{2}(L^\dagger L\rho_t + \rho_t L^\dagger L)\right) dt$$
$$+ \left(\frac{\vartheta\rho_t + \eta L\rho_t L^\dagger}{\vartheta + \eta \operatorname{Tr}\left(L\rho_t L^\dagger\right)} - \rho_t\right)\left(dy(t) - \left(\vartheta + \eta \operatorname{Tr}\left(L\rho_t L^\dagger\right)\right) dt\right)$$

- $dy(t) = 0$: $\rho_{t+dt} = \dfrac{K_0(\rho_t)}{\operatorname{Tr}\left(K_0(\rho_t)\right)}$ with probability $\operatorname{Tr}\left(K_0(\rho_t)\right)$
- $dy(t) = 1$: $\rho_{t+dt} = \dfrac{K_1(\rho_t)}{\operatorname{Tr}\left(K_1(\rho_t)\right)}$ with probability $\operatorname{Tr}\left(K_1(\rho_t)\right)$

$$K_0(\rho) = \left(M_0\rho M_0^\dagger + (1-\eta)L\rho L^\dagger dt\right)(1-\vartheta dt), \quad K_1(\rho) = \left(\vartheta\rho + \eta L\rho L^\dagger\right)\ dt$$
$$M_0 = I + \left(-\frac{i}{\hbar}H - \frac{1}{2}L^\dagger L\right) dt$$

Outline



P-controller (Markovian feedback[a]):
for $u(t) = f(y(t))$, average closed-loop dynamics of ρ remains governed by a Lindblad master equation.

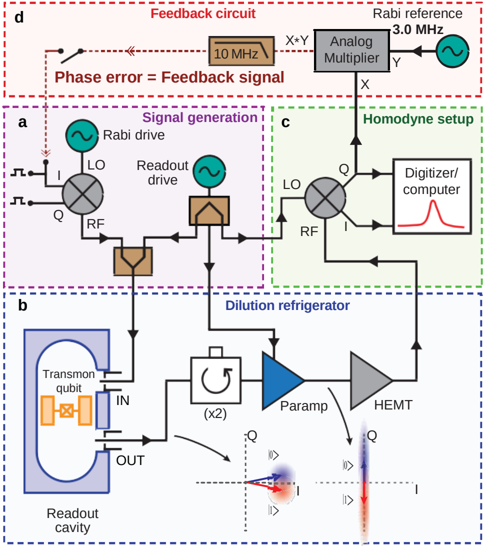
PID controller: no Lindblad master equation in closed-loop;

Nonlinear hidden-state stochastic systems: convergence analysis, Lyapunov exponents, dynamic output feedback, delays, robustness, ...

[a] H.M. Wiseman: Quantum Trajectories and Feedback. PhD Thesis, University of Queensland, 1994.

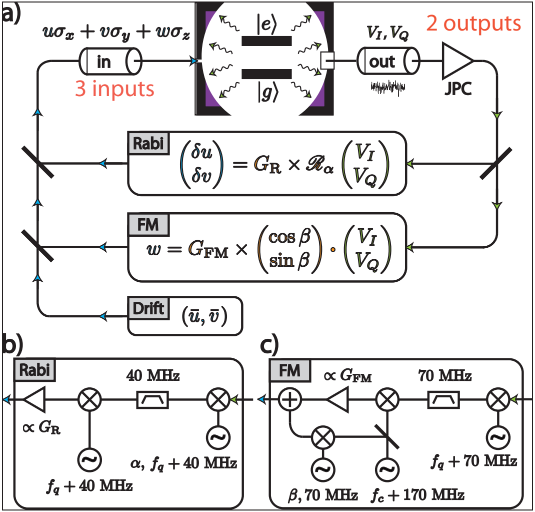
Short sampling times limit feedback complexity

First SISO measurement-based feedback for a superconducting qubit[5]



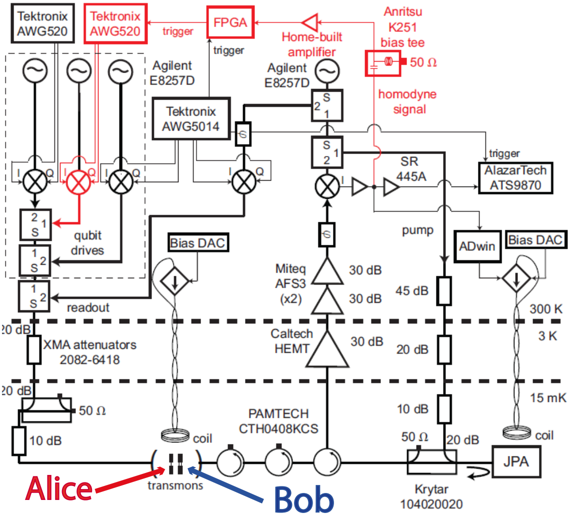
[5] R. Vijay, . . . , I. Siddiqi. Stabilizing Rabi oscillations in a superconducting qubit using quantum feedback. Nature 490, 77-80, October 2012.

First MIMO measurement-based feedback for a superconducting qubit[6]



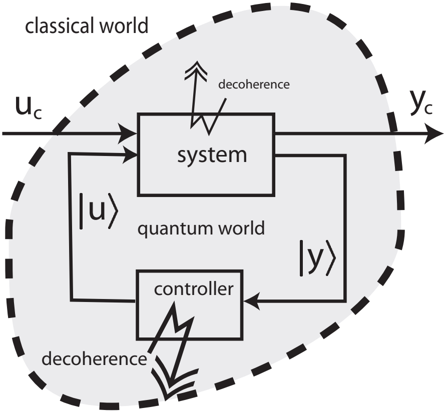
[6] P. Campagne-Ibarcq, . . . , B. Huard: Using Spontaneous Emission of a Qubit as a Resource for Feedback Control. Phys. Rev. Lett. 117(6), 2016.

Measurement-based feedback stabilization of $\frac{1}{\sqrt{2}}(|g\rangle \otimes |e\rangle + |e\rangle \otimes |g\rangle)$ [7]



[7] D. Ristè, . . . , L. DiCarlo: Deterministic entanglement of superconducting qubits by parity measurement and feedback. Nature 502, 350-354 (2013).

Quantum analogue of Watt speed governor: a **dissipative** mechanical system controls another mechanical system[8]



Optical pumping (Kastler 1950), coherent population trapping (Arimondo 1996)

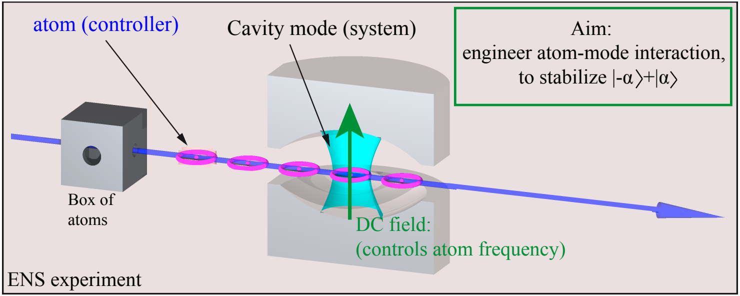
Dissipation engineering, autonomous feedback: (Zoller, Cirac, Wolf, Verstraete, Devoret, Schoelkopf, Siddiqi, Lloyd, Viola, Ticozzi, Mirrahimi, Sarlette, ...)

(S,L,H) theory and linear quantum systems: quantum feedback networks based on stochastic Schrödinger equation, Heisenberg picture (Gardiner, Yurke, Mabuchi, Genoni, Serafini, Milburn, Wiseman, Doherty, Gough, James, Petersen, Nurdin, Yamamoto, Zhang, Dong, . . .)

Stability analysis: Kraus maps and Lindblad propagators are always contractions (non commutative diffusion and consensus).

[8] J.C. Maxwell: On governors. Proc. of the Royal Society, No.100, 1868.

Reservoir engineering to stabilize "Schrödinger cats" $|\alpha\rangle + |-\alpha\rangle$ [9]



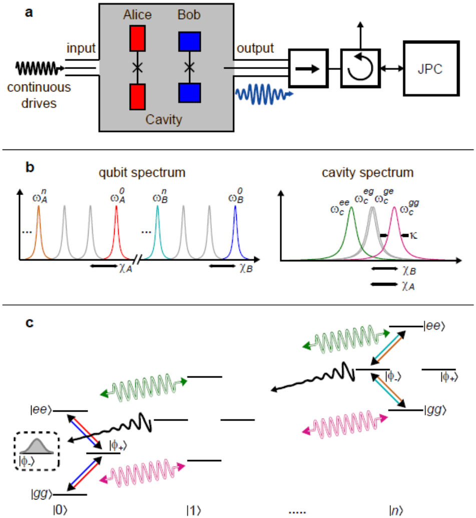
Jaynes-Cumming Hamiltionian

$$\boldsymbol{H}(t)/\hbar = \omega_c \boldsymbol{a}^\dagger \boldsymbol{a} \otimes \boldsymbol{I}_M + \boldsymbol{\omega_q(t)} \boldsymbol{I}_S \otimes \boldsymbol{\sigma_z}/2 + i\Omega(t)\left(\boldsymbol{a}^\dagger \otimes \boldsymbol{\sigma_-} - \boldsymbol{a} \otimes \boldsymbol{\sigma_+}\right)/2$$

with the open-loop control $t \mapsto \boldsymbol{\omega_q(t)}$ combining dispersive $\omega_q \neq \omega_c$ and resonant $\omega_q = \omega_c$ interactions.

Key issues: **convergence** of $\rho_{k+1} = \boldsymbol{K}(\rho_k) = \boldsymbol{M}_g \rho_k \boldsymbol{M}_g^\dagger + \boldsymbol{M}_e \rho_k \boldsymbol{M}_e^\dagger$.

[9] A. Sarlette et al: Stabilization of nonclassical states of the radiation field in a cavity by reservoir engineering. Phys. Rev. Lett. 107(1), 2011.



Lindblad master equation:

$$\frac{d}{dt}\rho = -i[H(t), \rho] + \kappa \mathcal{D}_a(\rho)$$
$$+ \frac{1}{T_1^A}\mathcal{D}_{\sigma_-^A}(\rho) + \frac{1}{2T_\phi^A}\mathcal{D}_{\sigma_z^A}(\rho)$$
$$+ \frac{1}{T_1^B}\mathcal{D}_{\sigma_-^B}(\rho) + \frac{1}{2T_\phi^B}\mathcal{D}_{\sigma_z^B}(\rho)$$

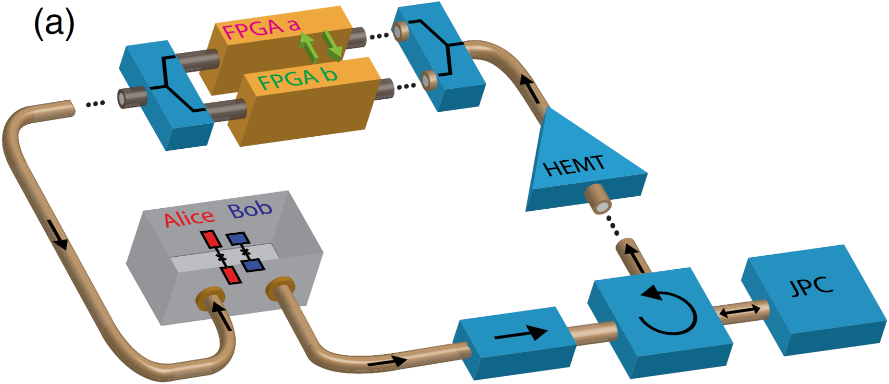
with

$$H(t)/\hbar = \left(\frac{\chi_A}{2}\sigma_z^A + \frac{\chi_B}{2}\sigma_z^B\right) a^\dagger a$$
$$+ 2\epsilon_c \cos\left(\frac{\chi_A+\chi_B}{2} t\right)\left(a + a^\dagger\right)$$
$$+ \Omega_0\left(\sigma_x^A + \sigma_x^B\right)$$
$$+\Omega_n\left(e^{-in\frac{\chi_A+\chi_B}{2}t}(\sigma_+^A - \sigma_+^B) + \text{h.c.}\right)$$

[10] S. Shankar, . . . , M.H. Devoret. Autonomously stabilized entanglement between two superconducting quantum bits. Nature, 504: 419-422, 2013.

Combining measurement-based feedback with dissipation engineering [11]

(a)



[11] Y. Liu, . . . , M.H. Devoret: Comparing and combining measurement-based and driven-dissipative entanglement stabilization. Phys. Rev. X 6, 2016.

Concluding remarks

- Three key quantum rules (highlighted by Haroche photon-box)

  1. Schrödinger equations defining unitary transformations,
  2. partial collapse of the wave packet: dissipation induced by measurement of observables with degenerate spectra,
  3. tensor product for composite systems,

  explain structure of **stochastic master equations**,
  illustrate importance of **spin/spring models.**

- Feedback control of **coherence** and **entanglement** in composite systems: important issues for quantum computing, simulation, metrology and communication.

- **Composite systems** : **curse of dimensionality** for quantum networks with delays in coherent feedback loops.

Importance of collaborations with experimental quantum physicists for defining relevant control engineering questions