Modeling and Control of Quantum Systems

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http://cas.ensmp.fr/~rouchon/QuantumSyst/index.html

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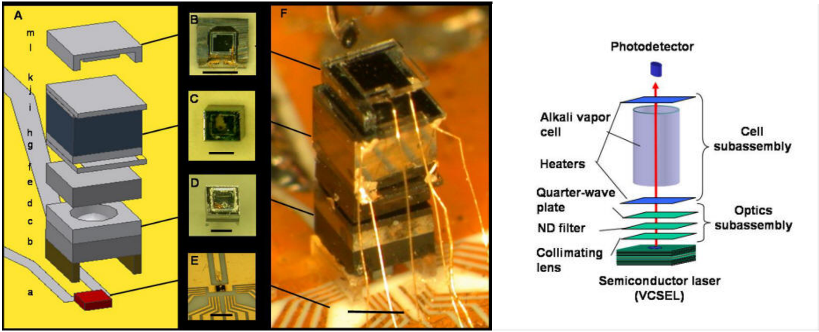
Outline

1. Chip-scale Atomic clock
   - The NIST MicroClock
   - The principle: Coherent Population Trapping
   - The system and its synchronization scheme
2. Convergence analysis
   - The open-loop stochastic differential equation
   - The closed-loop stochastic differential system
   - Sketch of convergence proof
3. Conclusion of the course

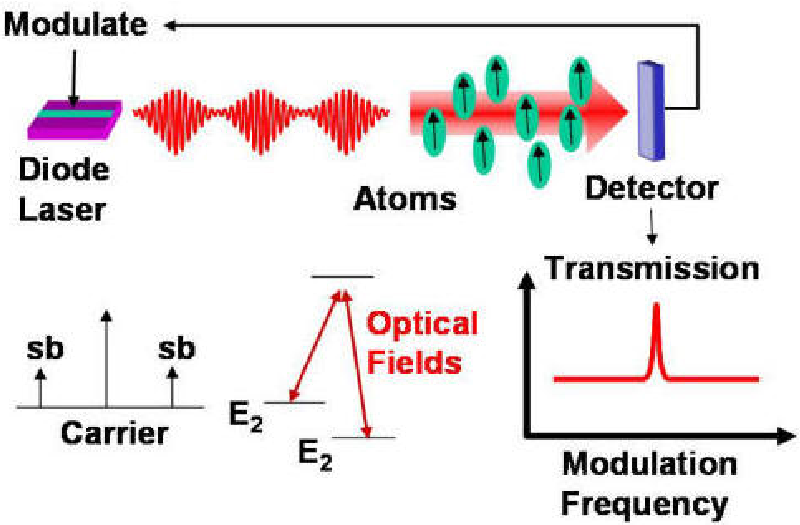
The NIST MicroClock[1]



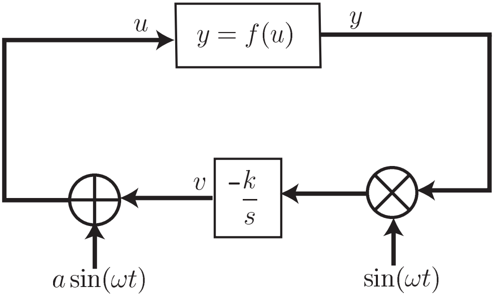
- Quartz crystal clocks: 1 second over few days.
- NIST chip-scale atomic clock: 1 second over 300 years
- High-Perf. atomic clocks: 1 second over 100 million years.

[1] NIST: National Institute of Standards and Technology, web-site: http://tf.nist.gov/timefreq/index.html.

The principle: Coherent Population Trapping[2]



[2] From the web-site: http://tf.nist.gov/timefreq/index.html.



Here $u = \omega_{diode}$ and $y = f(\omega_{diode})$ where f admits a sharp maximum at the unknown value $\bar{u} = \omega_{atom}$. $s = \frac{d}{dt}$, constant parameters (k, a, ω).

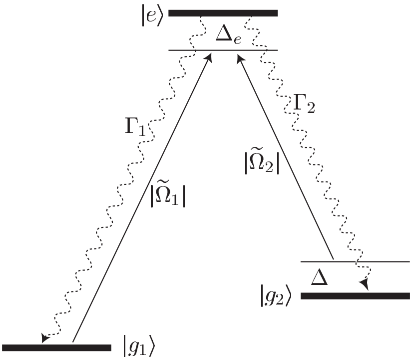
Extremum seeking via feedback: $u(t) = v(t) + a\sin(\omega t)$ where $v(t) \approx \omega_{atom}$ is adjusted via a dynamic time-varying output feedback (with $\omega, a, \sqrt{k} \ll \omega_{atom}$):

$$\frac{d}{dt} v(t) = -k\sin(\omega t)\ \overbrace{f\Big(\underbrace{v(t) + a\sin(\omega t)}_{u}\Big)}^{y}$$

This lecture describes a real-time synchronization scheme when the atomic cloud is replaced by a single atom[3].

[3]M-R, SIAM J. Control and Optimization, 2009.

The system and its synchronization scheme



Input: $\tilde{\Omega}_1, \tilde{\Omega}_2 \in \mathbb{C}$ and $u = \frac{d}{dt}\Delta$. Output: photo-detector click times corresponding to stochastic jumps from $|e\rangle$ to $|g_1\rangle$ or $|g_2\rangle$.

Synchronization goal: stabilize the unknown detuning Δ to 0.

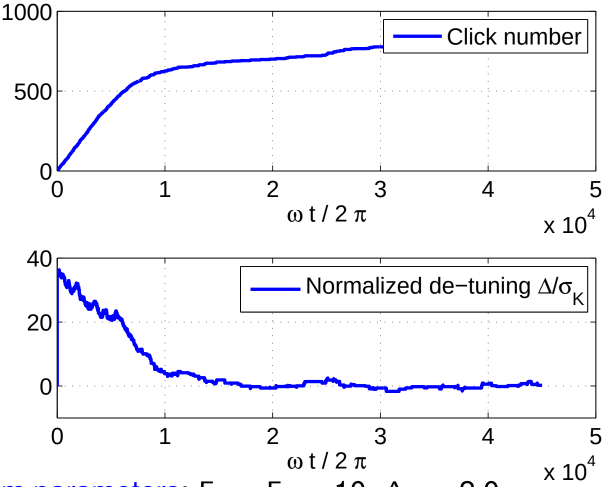
Two time-scales: $|\tilde{\Omega}_1|, |\tilde{\Omega}_2|, |\Delta_e|, |\Delta| \ll \Gamma_1, \Gamma_2$

Modulation of Rabi complex amplitudes $\tilde{\Omega}_1$ and $\tilde{\Omega}_2$:
$\tilde{\Omega}_1(t) = \Omega_1 - \imath\epsilon\Omega_2 \cos(\omega t)$, $\quad \tilde{\Omega}_2(t) = \imath\epsilon\Omega_1 \cos(\omega t) + \Omega_2$,
with $\Omega_1, \Omega_2 > 0$ constant, $\omega \ll \Gamma_1, \Gamma_2$ and $0 < \epsilon \ll 1$.

Detuning update $\quad \Delta_{N+1} = \Delta_N - K \frac{2\Omega_1\Omega_2}{\Omega_1^2+\Omega_2^2} \cos(\omega t_N)$

at each detected jump-time t_N. The gain $K > 0$ fixes the standard deviation σ_K: $\frac{16}{3}\sigma_K^2 = \epsilon K \frac{\Omega_1^2+\Omega_2^2}{\Gamma_1+\Gamma_2}$.

Closed-loop quantum trajectory (Matlab M-file: `SynchroCPT.m`)

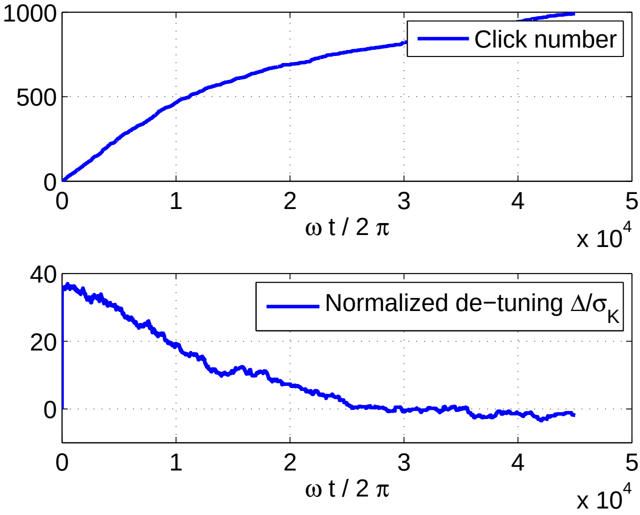


Λ-system parameters: $\Gamma_1 = \Gamma_2 = 10$, $\Delta_e = 2.0$

Modulation parameters: $\Omega_1 = \Omega_2 = 1.0$, $\omega = 2.8$, $\epsilon = 0.14$

Feedback gain $K = 0.0023$ leading to a standard deviation $\sigma_K = 0.0057$

Robustness



Detector efficiency of 50%, wrong jump detection of 50%, feedback-loop delay of τ with $\omega\tau = \pi/4$.

The slow/fast master equation

Master equation of the Λ-system

$$\frac{d}{dt}\rho = -\imath[\tilde{H}, \rho] + \frac{1}{2}\sum_{j=1}^{2}(2Q_j\rho Q_j^\dagger - Q_j^\dagger Q_j\rho - \rho Q_j^\dagger Q_j),$$

with jump operators $Q_j = \sqrt{\Gamma_j}\,|g_j\rangle\langle e|$ and Hamiltonian

$$\tilde{H} = \frac{\Delta}{2}(|g_2\rangle\langle g_2| - |g_1\rangle\langle g_1|) + \left(\Delta_e + \frac{\Delta}{2}\right)(|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|)$$
$$+ \widetilde{\Omega}_1\,|g_1\rangle\langle e| + \widetilde{\Omega}_1^*\,|e\rangle\langle g_1| + \widetilde{\Omega}_2\,|g_2\rangle\langle e| + \widetilde{\Omega}_2^*\,|e\rangle\langle g_2|\,.$$

Since $|\tilde{\Omega}_1|, |\tilde{\Omega}_2|, |\Delta_e|, |\Delta| \ll \Gamma_1, \Gamma_2$ we have two time-scales: a fast exponential decay for "$|e\rangle$" and a slow evolution for "$(|g_1\rangle, |g_2\rangle)$".

The slow master equation

Geometric reduction via center manifold techniques[4] leads to a reduced master equation that is still of Lindblad type with a slow Hamiltonian H and slow jump operators L_j:

$$\frac{d}{dt}\rho = -\imath[H,\rho] + \frac{1}{2}\sum_{j=1}^{2}(2L_j\rho L_j^\dagger - L_j^\dagger L_j\rho - \rho L_j^\dagger L_j),$$

with $H = \frac{\Delta}{2}\sigma_z = \frac{\Delta(|g_2\rangle\langle g_2| - |g_1\rangle\langle g_1|)}{2}$ and $L_j = \sqrt{\tilde{\gamma}_j}\,|g_j\rangle\langle b_{\tilde{\Omega}}|$ and where $\tilde{\gamma}_j = 4\frac{|\widetilde{\Omega}_1|^2+|\widetilde{\Omega}_2|^2}{(\Gamma_1+\Gamma_2)^2}\Gamma_j$ and $|b_{\tilde{\Omega}}\rangle$ is the bright state:

$$|b_{\tilde{\Omega}}\rangle = \frac{\widetilde{\Omega}_1}{\sqrt{|\widetilde{\Omega}_1|^2+|\widetilde{\Omega}_2|^2}}|g_1\rangle + \frac{\widetilde{\Omega}_2}{\sqrt{|\widetilde{\Omega}_1|^2+|\widetilde{\Omega}_2|^2}}|g_2\rangle$$

For $\Delta = 0$, ρ converges towards the dark state $|d_{\tilde{\Omega}}\rangle\langle d_{\tilde{\Omega}}|$:

$$|d_{\tilde{\Omega}}\rangle = -\frac{\widetilde{\Omega}_2^*}{\sqrt{|\widetilde{\Omega}_1|^2+|\widetilde{\Omega}_2|^2}}|g_1\rangle + \frac{\widetilde{\Omega}_1^*}{\sqrt{|\widetilde{\Omega}_1|^2+|\widetilde{\Omega}_2|^2}}|g_2\rangle.$$

[4] M-R 2009, IEEE-AC.

The stochastic differential slow model with the synchronization feedback

The reduced density matrix ρ obeys to

$$d\rho = -\imath\frac{\Delta}{2}[\sigma_z,\rho]\,dt + \left(\widetilde{\gamma}\left\langle b_{\widetilde{\Omega}}\right|\rho\left|b_{\widetilde{\Omega}}\right\rangle\rho\right)\,dt$$
$$-\frac{\widetilde{\gamma}}{2}\left(\rho\left|b_{\widetilde{\Omega}}\right\rangle\left\langle b_{\widetilde{\Omega}}\right| + \left|b_{\widetilde{\Omega}}\right\rangle\left\langle b_{\widetilde{\Omega}}\right|\rho\right)\,dt$$
$$+\left(\left|g_1\right\rangle\left\langle g_1\right| - \rho\right)dN_t^1 + \left(\left|g_2\right\rangle\left\langle g_2\right| - \rho\right)dN_t^2$$

$$d\Delta = K\frac{2\Omega_1\Omega_2}{\Omega_1^2+\Omega_2^2}\cos(\omega t)(dN_t^1 + dN_t^2) + \text{saturation at } \pm\frac{\gamma}{2}$$

with

$$\mathbb{E}\left(dN_t^1\right) = \widetilde{\gamma}_1 \operatorname{Tr}\left(\left|b_{\widetilde{\Omega}}\right\rangle\left\langle b_{\widetilde{\Omega}}\right|\rho\right)\,dt,$$
$$\mathbb{E}\left(dN_t^2\right) = \widetilde{\gamma}_2 \operatorname{Tr}\left(\left|b_{\widetilde{\Omega}}\right\rangle\left\langle b_{\widetilde{\Omega}}\right|\rho\right)\,dt$$

and $\widetilde{\Omega}_1(t) = \Omega_1 - \imath\epsilon\Omega_2\cos(\omega t)$, $\widetilde{\Omega}_2(t) = \imath\epsilon\Omega_1\cos(\omega t) + \Omega_2$

A convergence result

Claim

Take the above stochastic differential system with state ρ and Δ. Assume that the angle $\alpha = \arg(\Omega_1 + \imath\Omega_2)$ belongs to $]0, \frac{\pi}{2}[$. Then for sufficiently small ϵ and K, for sufficiently large ω,

$$\lim_{N\to\infty} \mathbb{E}\left(\Delta_N\right) = 0,$$

and

$$\limsup_{N\to\infty} \mathbb{E}\left(\Delta_N^2\right) \leq O(\epsilon^2).$$

Corollary

One has

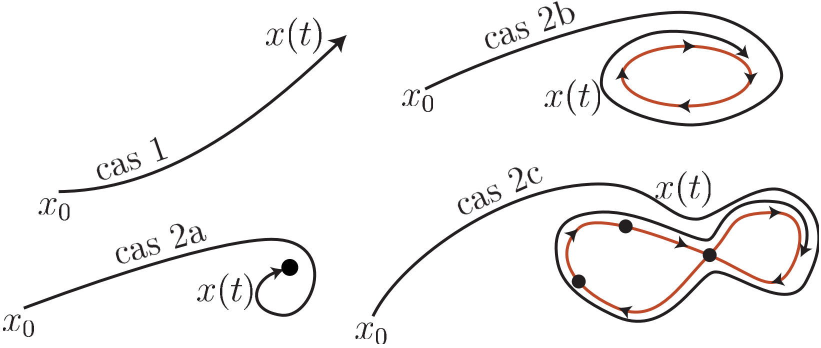
$$\limsup_{N\to\infty} \mathbb{P}\left(|\Delta_N| > \sqrt{\epsilon}\right) \leq O(\epsilon).$$

Steps of the convergence analysis

1. We start by analyzing the asymptotic behavior of the no-jump dynamics. We prove that the trajectories of the no-jump dynamics converge towards a unique small limit cycle around the dark state (Poincaré Bendixon theory).
2. This gives the asymptotic probability distribution of the jump times which will be a periodic function of time.
3. We will compute the conditional evolution of the expectation value of the detuning and its square. We will see that this evolution induces a **contraction** and we have the proof.

Poincaré-Bendixon theory

There are 4 types of asymptotic behaviors for a trajectory of an ordinary differential system $\frac{d}{dt}x = v(x)$ where x belongs to $\mathbb{R}^2$ or $\mathbb{S}^2 \sim \mathbb{R}^2 \cup \{\infty\}$.



Single frequency averaging ($0 \leq \varepsilon \ll 1$)

Take the perturbed system

$$\frac{dx}{dt} = \varepsilon f(x, t, \varepsilon)$$

with f smooth T-periodic versus t. Then exists a change of variables

$$x = z + \varepsilon w(z, t)$$

with w smooth and T-periodic versus t, such that

$$\frac{dz}{dt} = \varepsilon \bar{f}(z) + \varepsilon^2 f_1(z, t, \varepsilon)$$

where

$$\bar{f}(z) = \frac{1}{T} \int_0^T f(z, t, 0)\, dt$$

and f_1 smooth and T-periodic versus t
The average system reads: $\frac{d}{dt} z = \varepsilon \bar{f}(z)$.

Single frequency averaging (end)

- if $x(t)$ and $z(t)$ are, respectively, solutions of the perturbed and average systems, with initial conditions x_0 and z_0 such that $\|x_0 - z_0\| = O(\varepsilon)$, then $\|x(t) - z(t)\| = O(\varepsilon)$ on a time-interval of length of order $1/\varepsilon$.
- If $\overline{z}$ is an hyperbolic equilibrium of the average system, then exists $\overline{\varepsilon} > 0$ such that, for all $\varepsilon \in]0, \overline{\varepsilon}]$, the perturbed system admits a unique hyperbolic periodic orbit $\gamma_\varepsilon(t)$, close to $\overline{z}$, $\gamma_\varepsilon(t) = \overline{z} + O(\varepsilon)$, that could be reduced to a point, with a stability similar to those of $\overline{z}$ [5].
- In particular, if $\overline{z}$ is asymptotically stable, then γ_ε is also asymptotically stable and the approximation, up to $O(\varepsilon)$, of the trajectories of the perturbed system by those of the average ones is valid for $t \in [0, +\infty[$.

[5] The number of characteristic multipliers of γ_ε with modulus > 1 (resp. < 1) is equal to the number of characteristic exponents of $\overline{z}$ with real part > 0 (resp. < 0).

Quantum trajectories

In the absence of the quantum jumps, ρ evolves on the Bloch sphere according to ($\tilde{\gamma} = 4\frac{|\widetilde{\Omega}_1|^2+|\widetilde{\Omega}_2|^2}{\Gamma_1+\Gamma_2}$)

$$\frac{1}{\tilde{\gamma}}\frac{d}{dt}\rho = -\imath\frac{\Delta}{2\tilde{\gamma}}[\sigma_z,\rho] - \frac{\left|b_{\widetilde{\Omega}}\right\rangle\left\langle b_{\widetilde{\Omega}}\right|\rho + \rho\left|b_{\widetilde{\Omega}}\right\rangle\left\langle b_{\widetilde{\Omega}}\right|}{2} + \left\langle b_{\widetilde{\Omega}}\right|\rho\left|b_{\widetilde{\Omega}}\right\rangle\rho.$$

At each time step dt, ρ may jump towards the state $|g_1\rangle\langle g_1|$ or $|g_2\rangle\langle g_2|$ with a jump probability given by:

$$P_{jump}\,dt = \left(\tilde{\gamma}\,\left\langle b_{\widetilde{\Omega}}\right|\rho\left|b_{\widetilde{\Omega}}\right\rangle\right)\quad dt$$

Since $\tilde{\Omega}_1(t) = \Omega_1 - \imath\epsilon\Omega_2\cos(\omega t)$ and $\tilde{\Omega}_2(t) = \imath\epsilon\Omega_1\cos(\omega t) + \Omega_2$,

$$\tilde{\gamma}\left|b_{\widetilde{\Omega}}\right\rangle\left\langle b_{\widetilde{\Omega}}\right| = \gamma\left(|b\rangle + \imath\epsilon\cos(\omega t)\,|d\rangle\right)\left(\langle b| - \imath\epsilon\cos(\omega t)\,\langle d|\right)$$

with $\gamma = 4\frac{|\Omega_1|^2+|\Omega_2|^2}{\Gamma_1+\Gamma_2}$, $|b\rangle = \frac{\Omega_1|g_1\rangle+\Omega_2|g_2\rangle}{\sqrt{\Omega_1^2+\Omega_2^2}}$ and $|d\rangle = \frac{-\Omega_2|g_1\rangle+\Omega_1|g_2\rangle}{\sqrt{\Omega_1^2+\Omega_2^2}}$

With $\beta = 2\arg(\Omega_1 + \imath\Omega_2) = 2\alpha$ and
$\rho = \frac{1+X(|b\rangle\langle d|+|d\rangle\langle b|)+Y(\imath|b\rangle\langle d|-\imath|d\rangle\langle b|)+Z(|d\rangle\langle d|-|b\rangle\langle b|)}{2}$:

$$\frac{d}{dt}X = -\Delta\cos\beta Y - \gamma\left(\epsilon\cos(\omega t)Y + \frac{1-\epsilon^2\cos^2(\omega t)}{2}Z\right)X$$

$$\frac{d}{dt}Y = \Delta\cos\beta X - \Delta\sin\beta Z + \gamma\epsilon\cos(\omega t) - \gamma\left(\epsilon\cos(\omega t)Y + \frac{1-\epsilon^2\cos^2(\omega t)}{2}Z\right)Y$$

$$\frac{d}{dt}Z = \Delta\sin\beta Y + \gamma\left(\frac{1-\epsilon^2\cos^2(\omega t)}{2}\right) - \gamma\left(\epsilon\cos(\omega t)Y + \frac{1-\epsilon^2\cos^2(\omega t)}{2}Z\right)Z$$

The jump probability per unit of time is

$$P_{jump} = \frac{\gamma}{2}\left(1 - Z - 2\epsilon\cos(\omega t)Y + \epsilon^2\cos^2(\omega t)(1+Z)\right).$$

Just after a jump (X, Y, Z) is reset to $\pm(\sin\beta, 0, \cos\beta)$.

Convergence of the no-jump dynamics

$$\frac{d}{dt}X = -\Delta\cos\beta Y - \gamma\left(\epsilon\cos(\omega t)Y + \frac{1-\epsilon^2\cos^2(\omega t)}{2}Z\right)X$$

$$\frac{d}{dt}Y = \Delta\cos\beta X - \Delta\sin\beta Z + \gamma\epsilon\cos(\omega t) - \gamma\left(\epsilon\cos(\omega t)Y + \frac{1-\epsilon^2\cos^2(\omega t)}{2}Z\right)Y$$

$$\frac{d}{dt}Z = \Delta\sin\beta Y + \gamma\left(\frac{1-\epsilon^2\cos^2(\omega t)}{2}\right) - \gamma\left(\epsilon\cos(\omega t)Y + \frac{1-\epsilon^2\cos^2(\omega t)}{2}Z\right)Z$$

For $|\Delta| < \frac{\gamma}{2}$ and $0 < \epsilon \ll 1$, the above time-periodic nonlinear system admits a quasi-global asymptotically stable periodic orbit (proof: Poincaré-Bendixon with $\epsilon = 0$ and averaging using $\omega \gg \gamma$).
This periodic orbit reads

$$(X, Y, Z) = \left(0 \quad , \quad -2\sin\beta\frac{\Delta}{\gamma} + \frac{2\gamma^2\cos(\omega t)+4\gamma\omega\sin(\omega t)}{4\omega^2+\gamma^2}\epsilon \quad , \quad 1\right)$$

up to second order terms in ϵ and $\frac{\Delta}{\gamma}$.

When $\omega \gg \gamma$, $P_{jump} \approx \gamma\left(\epsilon\cos(\omega t) + \frac{\Delta\sin\beta}{\gamma}\right)^2$ if the last jump occurs more that few $-\log\epsilon/\gamma$ second(s) ago.[6]

[6] Replace Z by $1 - \frac{X^2+Y^2}{2}$ in previous formula giving P_{jump}.

Detuning update as a discrete-time stochastic process

Our analysis neglects the transient just after a jump.
When a jump occurs at t_N, we have

$$\Delta_{N+1} = \Delta_N - K \sin\beta \cos(\omega t_N)$$

and its probability was proportional to $\left(\epsilon \cos(\omega t_N) + \frac{\Delta_N \sin\beta}{\gamma}\right)^2$.
The phase $\varphi = \omega t_N$ can be seen as a stochastic variable in $[0, 2\pi]$ with the following probability density $P_{\Delta_N}(\varphi)$ on $[0, 2\pi]$:

$$P_{\Delta_N}(\varphi) = \frac{\left(\epsilon \cos(\varphi) + \frac{\Delta_N \sin\beta}{\gamma}\right)^2}{2\pi \left(\frac{\epsilon^2}{2} + \frac{\Delta_N^2 \sin^2\beta}{\gamma^2}\right)}$$

The de-tuning update is thus a discrete-time stochastic process

$$\Delta_{N+1} = \Delta_N - K \sin\beta \cos\varphi$$

where the probability of $\varphi \in [0, 2\pi]$ depends on Δ_N.

We assume here $|\Delta| \ll \epsilon\gamma$ (remember $\gamma \ll \omega \ll \Gamma_1 + \Gamma_2$):

$$\Delta_{N+1} = \Delta_N - K \sin\beta \cos\varphi$$

with φ of probability density $P_{\Delta_N}(\varphi) \approx \frac{\cos^2\varphi}{\pi} + 2\frac{\Delta_N \sin\beta}{\pi\epsilon\gamma}\cos\varphi$.
Simple computations yield to[7]

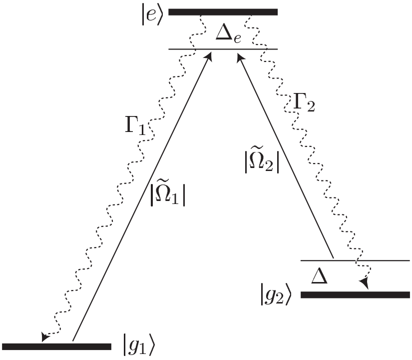
$$\mathbb{E}\left(\Delta_{N+1} \;/\; \Delta_N\right) = \left(1 - \frac{2K\sin^2\beta}{\epsilon\gamma}\right)\Delta_N$$

For $0 < K \leq \frac{\epsilon\gamma}{\sin^2\beta}$, $E(\Delta_N)$ tends to zero.
Similarly, we have

$$\mathbb{E}\left(\Delta^2_{N+1} \;/\; \Delta_N\right) = \left(1 - \frac{4K\sin^2\beta}{\epsilon\gamma}\right)\Delta^2_N + \frac{3K^2\sin^2\beta}{8}$$

For $0 < K \leq \frac{\epsilon\gamma}{2\sin^2\beta}$, $E(\Delta^2_N)$ converges to $\sigma^2_K = \frac{3\epsilon\gamma K}{32}$.

[7] $\mathbb{E}\left(\Delta_{N+1} \;/\; \Delta_N\right)$ stands for the conditional expectation-value of Δ_{N+1} knowing Δ_N.



Rabi frequency modulations:

$\tilde{\Omega}_1(t) = \Omega_1 - \imath\epsilon\Omega_2 \cos(\omega t)$

$\tilde{\Omega}_2(t) = \imath\epsilon\Omega_1 \cos(\omega t) + \Omega_2$

with $\Omega_1, \Omega_2 \ll \Gamma = \Gamma_1 + \Gamma_2$,

$0 < \epsilon \ll 1$ and

$\frac{\Omega_1^2+\Omega_2^2}{\Gamma_1+\Gamma_2} = \gamma \ll \omega \ll \Gamma$

Detuning update

$\Delta_{N+1} = \Delta_N - K \sin\beta \cos(\omega t_N)$

with $K > 0$, $\beta = 2\arg(\Omega_1 + \imath\Omega_2)$.

A discrete-time stochastic process where the gain $K > 0$ drives

- the convergence speed with a contraction of $\left(1 - \frac{2K\sin^2\beta}{\epsilon\gamma}\right)$ for $E(\Delta_N)$ at each iteration
- the precision via the asymptotic standard deviation $\sigma_K = \frac{\sqrt{3\epsilon\gamma K}}{4\sqrt{2}}$.

Conservative deterministic models for closed-quantum systems

Conservative models (Schrödinger, closed-quantum systems):

$$i \frac{d}{dt} |\psi\rangle = H |\psi\rangle, \quad \frac{d}{dt}\rho = -i[H, \rho]$$

showing that $|\psi\rangle_t = U_t |\psi\rangle_0$ and $\rho_t = U_t \rho_0 U_t^\dagger$ with propagator U_t defined by $i \frac{d}{dt} U = HU, \; U_0 = \mathbf{1}$.

- $H = H_0 + \sum_k u_k H_k$: controllability (Lie algebra in finite dimension, importance of the spectrum in infinite dimension, Law-Eberly method), optimal control (minimum time in finite dimension only).
- Widely used motion planing based on two approximations: RWA; adiabatic invariance (robustness).
- Non commutative calculus with operators (Bra, Ket and Dirac notations).
- Key issues attached to composite systems (tensor product). Two classes of important subsystems: finite-dimensional ones (2-level, Bloch sphere, Pauli matrices); infinite dimensional ones (harmonic oscillator, annihilation operator).

Dissipative models for open-quantum systems (1)

Discrete-time models are Markov chains

$$\rho_{k+1} = \frac{1}{p_\nu(\rho_k)} M_\nu \rho_k M_\nu^\dagger \quad \text{with proba.} \quad p_\nu(\rho_k) = \text{Tr}\left(M_\nu \rho_k M_\nu^\dagger\right)$$

associated to Kraus maps (ensemble average, open-quantum channel maps)

$$\mathbb{E}\left(\rho_{k+1}/\rho_k\right) = K(\rho_k) = \sum_\nu M_\nu \rho_k M_\nu^\dagger \quad \text{with} \quad \sum_\nu M_\nu^\dagger M_\nu = \mathbf{1}$$

Continuous-time models are stochastic differential systems

$$d\rho = -i[H,\rho]dt$$
$$+ \sum_\nu \text{Tr}\left(L_\nu \rho L_\nu^\dagger\right)\rho dt - \tfrac{1}{2}(L_\nu^\dagger L_\nu \rho + \rho L_\nu^\dagger L_\nu)dt + \left(\frac{L_\nu \rho L_\nu^\dagger}{\text{Tr}\left(L_\nu \rho L_\nu^\dagger\right)} - \rho\right) dN_t^\nu$$

driven by Poisson processes dN_t^ν with $\mathbb{E}\left(dN_t^\nu\right) = \text{Tr}\left(L_\nu \rho L_\nu^\dagger\right) dt$ (possible approximations by Wiener processes) and associated to Lindbald master equations:

$$\tfrac{d}{dt}\rho = -i[H,\rho] + \tfrac{1}{2}\sum_\nu \left(2L_\nu \rho L_\nu^\dagger - L_\nu^\dagger L_\nu \rho - \rho L_\nu^\dagger L_\nu\right),$$

Dissipative models for open-quantum systems (2)

Ensemble and average dynamics (Kraus maps (discrete-time) or Lindbald equations (continuous-time)):

- Stability induces by contraction (nuclear norm or fidelity).
- Decoherence free spaces: Ω-limits are affine spaces; they can be reduced to a point (pointer-states); design of M_ν and L_ν to achieve convergence towards prescribed affine spaces (reservoir engineering, QND measurements, ...).

Lindbald partial differential equation for the density operator $\rho(x, y)$, $(x, y) \in \mathbb{R}^2$,

$$\frac{d}{dt}\rho = \overbrace{[ua^\dagger - u^* a, \rho]}^{\text{Schrödinger}} + \overbrace{\gamma(n_{th} + 1)\mathbb{D}[a](\rho)}^{\text{cavity decay}} + + \overbrace{\gamma n_{th}\mathbb{D}[a^\dagger](\rho)}^{\text{thermal photon}}$$

where $\mathbb{D}[L](\rho) = \left(L\rho L^\dagger - \frac{L^\dagger L\rho + \rho L^\dagger L}{2}\right)$. It describes a quantized field trapped inside a finite fitness cavity (decay time $1/\gamma$), subject to a coherent excitation of amplitude $u \in \mathbb{C}$ and an incoherent coupling to a thermal field with $n_{th} \geq 0$ average photons .

Markov chain (discrete-time) or SDE (continuous time):

- Quantum filters provides $\hat{\rho}$, a real-time estimation of the state ρ based on measurements outcomes (in the ideal case $F(\rho, \hat{\rho})$ is sub-martingale).
- Feedback stabilization towards a goal pure state $\bar{\rho}$: $u(\rho)$ based on Lyapunov function $\text{Tr}(\bar{\rho}, \rho) = F(\bar{\rho}, \rho)$.
- Quantum separation principle always works for $u(\hat{\rho})$ in case of global convergence with feedback $u(\rho)$.
- Coherent feedback scheme: the controller is also a quantum system (not a classical one as above).