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KKL observer design for sensorless induction motors

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***Abstract*— We propose a novel observer for speed and torque estimation in induction motors, using only electrical measurements and assuming the parameters known. The design is based on a novel representation of the motor model taking the form of a cascade of a flux subsystem and a velocity subsystem with known injection terms. After giving sufficient conditions for the uniform strong observability of those subsystems, we exploit the KKL approach to design a globally asymptotically stable observer without relying on any time-scale separation. We provide a method to optimize the observer parameters and illustrate its performance on simulation in a realistic scenario.**

I. Introduction

Estimating the velocity, load torque and flux of induction motors from the voltages and currents is an important issue. It is in particular a key step in "sensorless" control algorithms of variable frequency drives, see e.g. [1], [2]. Popular observers for the induction motors include Luenberger observers [3], Extended Kalman Filters [4] or sliding mode observers [5], [6]. In this paper, we propose another observer relying on a novel representation of the motor model, made of a cascade of two subsystems with known input terms. This representation allows the design of globally exponentially stable observers and does not require any two-time-scale assumption. Indeed, some of the contributions cited above assume that the stator current is "fast" while the rotor velocity and flux are "slow". But this approximation can be rather crude, especially when it is enforced by a fast current loop as in a variable fequency drive [7]. While in a companion paper [8] we exploit this new model representation with Kalman observers from a practical point of view, we present here another route based on the "*Kazantzis-Kravaris-Luenberger*" (KKL) methodology developped in [9], [10]. We demonstrate the global asymptotic convergence of a KKL observer based on *uniform strong differential observability*, for which we provide sufficient conditions. We also provide a method to optimize the observer parameters to increase its robustness, a relatively absent issue in the KKL literature sofar.

The standard equations of the induction motor assuming no magnetic saturation read in the general xy frame (see for instance [11, Chapter 6])

$$\frac{\mathrm{d}\psi_s^{xy}}{\mathrm{d}t} + R_s\imath_s^{xy} = u_s^{xy} - \omega_e\mathcal{J}\psi_s^{xy} \qquad \text{(voltage law for stator)}$$

$$\frac{\mathrm{d}\tilde\psi_r^{xy}}{\mathrm{d}t} + R_r\tilde\imath_r^{xy} = (\omega_r - \omega_e)\mathcal{J}\tilde\psi_r^{xy} \qquad \text{(voltage law for rotor)}$$

$$\psi_s^{xy} = L_s\imath_s^{xy} + L_m\tilde\imath_r^{xy} \qquad \text{(flux-current relation)}$$

$$\tilde\psi_r^{xy} = L_m\imath_s^{xy} + L_r\tilde\imath_r^{xy} \qquad \text{(flux-current relation)}$$

$$\frac{J}{n}\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = T_{em} - T_l \qquad \text{(mechanical equation)}$$

$$T_{em} = nL_m\langle \imath_s^{xy}, \mathcal{J}\tilde\imath_r^{xy}\rangle \qquad \text{(electromagnetic torque).}$$

where ψ_s^{xy} (stator flux), $\tilde\psi_r^{xy}$ (rotor flux referred to stator), $\imath_s^{xy}$ (stator current), $\tilde\imath_r^{xy}$ (rotor current referred to stator) and u_s^{xy} (voltage impressed to stator) are 2×1 vectors; ω_r (rotor velocity), ω_e (arbitrary frame velocity), T_{em} (electromagnetic torque) and T_l (load torque) are scalars; $R_s, R_r, L_s, L_r, L_m, J, n$ are constant (or possibly slowly-varying) parameters; $\mathcal{I} := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathcal{J} := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$; finally, $\langle\cdot,\cdot\rangle$ denotes the dot product. The physical control input is $u_s^{\alpha\beta} = \mathcal{R}(\theta_e)u_s^{xy}$, where the frame angle θ_e is defined by $\frac{\mathrm{d}\theta_e}{\mathrm{d}t} := \omega_e$, and where $\mathcal{R}(\theta_e) := \begin{pmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{pmatrix}$ is the rotation matrix with angle θ_e. Being chosen at will, the frame velocity ω_e can thus also be seen as a control input. In particular, in standard terms, the choice $\omega_e := 0$ and $xy := \alpha\beta$ corresponds to the $\alpha\beta$ frame, whereas $\omega_e := \omega_s$ with ω_s the stator velocity, and $xy := dq$ corresponds to the dq frame. Any of those standard frames is eligible for observer design because ω_e and thus all the corresponding current and voltage signals are known. As the disturbance torque T_l is unknown, we model it in the simplest way as a constant, i.e. we supplement the previous equations with

$$\frac{\mathrm{d}T_l}{\mathrm{d}t} = 0 \quad \text{(load torque model).}$$

We assume u_s^{xy} (as the known control input or as a measurement), ω_e (as the frame velocity), $\imath_s^{xy}$ (as a measurement), and all the parameters are known. We seek an asymptotic estimation of all the unknown variables, in particular ω_r and T_l using only these known signals. We start by deriving a cascaded state form in Section II made of a *flux subsystem* and a *velocity subsystem*. After describing the operating conditions providing differential observability of each subsystem in Section III, we build first a KKL observer estimating ψ_s^{xy} in Section IV, and then a KKL observer estimating ω_r and T_l, *knowing* ψ_s^{xy}, in Section V. Finally, we show how to optimize the observer parameters in Section VI and illustrate the performance on simulation in a realistic scenario in Section VII.

II. A state form with output injection terms

To design our observer, the first step is to rewrite the model of the induction motor as a system in state form. The first contribution of this paper is to show that the system can be

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decomposed into a cascade of two subsystems with output injection terms that is appropriate for observer design:

- one independent from ω_r and T_l, allowing to estimate ψ_s^{xy} from the knowledge of $\imath_s^{xy}$, u_s^{xy} and ω_e;
- one allowing to estimate ω_r and T_l from the knowledge of $\imath_s^{xy}$ and u_s^{xy}, as well as the knowledge of ψ_s^{xy} obtained from the first system.

To this end, we choose as state variables $\psi_s^{xy}, \omega_r, T_l$ and the two orthogonal projections of $\imath_s^{xy}$ defined by

$$\jmath_s := \langle \imath_s^{xy}, \psi_s^{xy} - \tfrac{\sigma L_s}{2}\imath_s^{xy}\rangle$$
$$\tau := \langle \imath_s^{xy}, \mathcal{J}(\psi_s^{xy} - \tfrac{\sigma L_s}{2}\imath_s^{xy})\rangle = \langle \imath_s^{xy}, \mathcal{J}\psi_s^{xy}\rangle$$

Notice τ is the (scaled) electromagnetic torque. Indeed,

$$\tau = \langle \imath_s^{xy}, \mathcal{J}\psi_s^{xy}\rangle = \langle \imath_s^{xy}, \mathcal{J}(L_s\imath_s^{xy} + L_m\tilde{\imath}_r^{xy})\rangle = \tfrac{1}{n}T_{em}.$$

Using the equations of the model, it can be proved that

$$\frac{\mathrm{d}\psi_s^{xy}}{\mathrm{d}t} = u_s^{xy} - R_s\imath_s^{xy} - \omega_e\mathcal{J}\psi_s^{xy} \tag{1a}$$
$$\sigma L_s\frac{\mathrm{d}\jmath_s}{\mathrm{d}t} = \tfrac{R_r}{L_r}|\psi_s^{xy}|^2 - R_+\jmath_s + \langle\psi_s^{xy}, u_s^{xy}\rangle - \tfrac{\sigma L_s}{2}R_-|\imath_s^{xy}|^2 \tag{1b}$$
$$\sigma L_s\frac{\mathrm{d}\tau}{\mathrm{d}t} = -R_t\tau - \langle\psi_s^{xy} - \sigma L_s\imath_s^{xy}, \omega_r\psi_s^{xy} + \mathcal{J}u_s^{xy}\rangle \tag{1c}$$
$$\frac{J}{n}\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = n\tau - T_l \tag{1d}$$
$$\frac{\mathrm{d}T_l}{\mathrm{d}t} = 0, \tag{1e}$$

where $R_t := R_s + R_r\frac{L_s}{L_r}$, $R_+ := R_s + R_r\frac{L_s}{L_r}(\sigma+1)$, $R_- := R_s + R_r\frac{L_s}{L_r}(\sigma-1)$, and $\sigma := 1 - \frac{L_m^2}{L_sL_r}$; notice the so-called leakage factor σ is usually small (typically at most 0.05), which means that the current-derived variables $\jmath_s, \tau$ are "fast" variables, whereas $\psi_s^{xy}, \omega_r, \tau$ are "slow" variables.

Finally, the measured output $\imath_s^{xy}$ translates as the two implicit output equations

$$0 = \jmath_s - \langle \imath_s^{xy}, \psi_s^{xy} - \tfrac{\sigma L_s}{2}\imath_s^{xy}\rangle$$
$$0 = \tau - \langle \imath_s^{xy}, \mathcal{J}\psi_s^{xy}\rangle.$$

Those equations can be considered as fictitious measurements of the state, known to be constantly equal to zero.

A difficulty in designing an observer directly from (1) comes from the polynomial dependence on the state components to be estimated (terms $|\psi_s^{xy}|^2$ and $\omega_r|\psi_s^{xy}|^2$). Our design relies on the fact that (1) can be seen as a cascaded system. On the one hand, the "flux subsystem"

$$\frac{\mathrm{d}\psi_s^{xy}}{\mathrm{d}t} = u_s^{xy} - R_s\imath_s^{xy} - \omega_e\mathcal{J}\psi_s^{xy} \tag{2a}$$
$$\sigma L_s\frac{\mathrm{d}\jmath_s}{\mathrm{d}t} = \tfrac{R_r}{L_r}|\psi_s^{xy}|^2 - R_+\jmath_s + \langle\psi_s^{xy}, u_s^{xy}\rangle - \tfrac{\sigma L_s}{2}R_-|\imath_s^{xy}|^2 \tag{2b}$$
$$0 = \jmath_s - \langle \imath_s^{xy}, \psi_s^{xy} - \tfrac{\sigma L_s}{2}\imath_s^{xy}\rangle, \tag{2c}$$

which is independent from τ, T_l and ω_r and which will be used to estimate ψ_s^{xy} via a so-called *flux observer*; on the other hand, the "velocity subsystem"

$$\sigma L_s\frac{\mathrm{d}\tau}{\mathrm{d}t} = -R_t\tau - \langle\psi_s^{xy} - \sigma L_s\imath_s^{xy}, \omega_r\psi_s^{xy} + \mathcal{J}u_s^{xy}\rangle \tag{3a}$$
$$\frac{J}{n}\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = n\tau - T_l \tag{3b}$$
$$\frac{\mathrm{d}T_l}{\mathrm{d}t} = 0 \tag{3c}$$

$$\langle \imath_s^{xy}, \mathcal{J}\psi_s^{xy}\rangle = \tau \tag{3d}$$

where not only $\imath_s^{xy}, u_s^{xy}$ but also ψ_s^{xy} are seen as known injection terms, exploiting the estimation of ψ_s^{xy} coming from the flux observer. This latter velocity subsystem is linear with injection and will be used to estimate ω_r and T_l via a so-called "velocity observer".

III. Observability

Before designing an observer, we study the observability of each subsystem from the knowledge of their corresponding inputs and outputs.

Definition 1: A system $\dot{x} = f(x,t)$ with output map $y = h(x,t)$ is *differentially observable* of order p if the map made of h and its $p-1$ first Lie-derivatives

$$H_p(x,t) = (h(x,t), L_{(f,1)}h(x,t), \ldots, L_{(f,1)}^{p-1}h(x,t))$$

is injective with respect to x for all $t \geq 0$, where we denote

$$L_{(f,1)}h(x,t) = \frac{\partial h}{\partial x}(x,t)f(x,t) + \frac{\partial h}{\partial t}(x,t)$$

and $L_{(f,1)}^{(k)}h(x,t) = L_{(f,1)}L_{(f,1)}^{(k-1)}h(x,t)$. Besides, it is *uniformly strongly differentially observable* of order p if there exists $\ell_H > 0$ such that for all (x_a, x_b) and all $t \geq 0$,

$$|H_p(x_a,t) - H_p(x_b,t)| \geq \ell_H|x_a - x_b| \ . \tag{4}$$

Differential observability means that the state is entirely determined at each time by the knowledge of the output and its successive time derivatives. Uniform strong differentially observability additionally requires that this injectivity be of Lipschitz type and uniform in time. This property allows to obtain convergence guarantees of KKL observers [9].

A. Differential observability of ψ_s^{xy}

Because the signals in a certain frame xy differ from other frames $x'y'$ by some known rotations, it is equivalent to study observability in some frame or another. However, some may be more or less handy depending on the context.

1) In $\alpha\beta$ frame: When no assumption is made on the regime of the motor, one may simply consider the nominal $\alpha\beta$ frame where $\omega_e = 0$. Differentiating twice the output of (2) with respect to time provides the following equations

$$\jmath_s = \left\langle \imath_s^{\alpha\beta}, \psi_s^{\alpha\beta} - \frac{\sigma L_s}{2}\imath_s^{\alpha\beta}\right\rangle$$
$$\frac{R_r}{L_r}|\psi_s^{\alpha\beta}|^2 + \langle\psi_s^{\alpha\beta}, v_1^{\alpha\beta}\rangle = v_0^{\alpha\beta}$$
$$\langle\psi_s^{\alpha\beta}, v_2^{\alpha\beta}\rangle = \frac{\mathrm{d}v_0}{\mathrm{d}t} - \langle u_s^{\alpha\beta} - R_s\imath_s^{\alpha\beta}, v_1^{\alpha\beta}\rangle$$

where the $v_i^{\alpha\beta}$ are known signals with in particular

$$v_1^{\alpha\beta} = u_s^{\alpha\beta} - R_+ \imath_s^{\alpha\beta} - \sigma L_s \frac{\mathrm{d}\imath_s^{\alpha\beta}}{\mathrm{d}t}$$
$$v_2^{\alpha\beta} = 2\frac{R_r}{L_r}(u_s^{\alpha\beta} - R_s \imath_s^{\alpha\beta}) + \frac{\mathrm{d}v_1^{\alpha\beta}}{\mathrm{d}t}$$

It follows that $\langle \psi_s^{\alpha\beta}, v_2^{\alpha\beta}\rangle$ is uniquely defined by the output and its two first derivatives. Continuing differentiating the output, we obtain $O_p \psi_s^{\alpha\beta}$ from the output and its $p-1$ first derivatives with

$$O_p := \begin{pmatrix} v_2^{\alpha\beta} & \frac{\mathrm{d}v_2^{\alpha\beta}}{\mathrm{d}t} & \dots & \frac{\mathrm{d}^{(p-3)} v_2^{\alpha\beta}}{\mathrm{d}t^{p-3}} \end{pmatrix}^\top .$$

Then, $\psi_s^{\alpha\beta}$, and thus $\jmath_s$ by the first equation, are differentially observable of order p if and only if O_p is full rank. By linearity of $O_p \psi_s^{\alpha\beta}$ with respect to ψ_s^{xy} and of the first equation with respect to $\jmath_s$, we then obtain (4) via the uniform full rank O_p as stated next.

Lemma 1: The flux subsystem (2) is uniformly strongly differentially observable of order $p \geq 4$ if and only if there exists $\ell > 0$ such that for all $t \geq 0$,

$$O_p(t)^\top O_p(t) \geq \ell \mathcal{I} .$$

2) Steady state in dq-frame: In the particular case of an induction motor at steady state, where $\imath_s^{dq}$, u_s^{dq} and ω_s are constant, it is relevant to investigate the meaning of the previously found condition. By successively differentiating the output in the dq-frame, we get this time

$$\jmath_s = \left\langle \imath_s^{dq}, \psi_s^{dq} - \frac{\sigma L_s}{2}\imath_s^{dq} \right\rangle$$
$$\frac{R_r}{L_r}|\psi_s^{dq}|^2 + \langle \psi_s^{dq}, v_1^{dq}\rangle = v_0^{dq}$$
$$\langle \psi_s^{dq}, v_2^{dq}\rangle - \omega_s \langle \mathcal{J}\psi_s^{dq}, v_1^{dq}\rangle = v_3^{dq}$$
$$-\omega_s^2 \langle \psi_s^{dq}, v_1^{dq}\rangle - \omega_s \langle \mathcal{J}\psi_s^{dq}, v_2^{dq}\rangle = v_4^{dq}$$

where the signals (v_i) are known with in particular

$$v_1^{dq} = u_s^{dq} - R_+ \imath_s^{dq} - \sigma L_s \omega_s \mathcal{J} \imath_s^{dq}$$
$$v_2^{dq} = 2\frac{R_r}{L_r}(u_s^{dq} - R_s \imath_s^{dq})$$

It follows that the quantity

$${\psi_s^{dq}}^\top \begin{pmatrix} v_2^{dq} + \omega_s \mathcal{J} v_1^{dq} & -{\omega_s}^2 v_1^{dq} + \omega_s \mathcal{J} v_2^{dq} \end{pmatrix}$$

is uniquely determined, so that in turn ψ_s^{dq} is uniquely determined if the determinant

$$(-{\omega_s}^2 v_1^{dq} + \omega_s \mathcal{J} v_2^{dq})^\top \mathcal{J}(v_2^{dq} + \omega_s \mathcal{J} v_1^{dq}) = \omega_s \|\omega_s v_1^{dq} - \mathcal{J} v_2^{dq}\|^2$$

is non zero. This proves the following lemma.

Lemma 2: If $\imath_s^{dq}$, u_s^{dq} and ω_s are constant without $\imath_s^{dq} = u_s^{dq} = 0$, the flux subsystem (2) is uniformly strongly differentially observable of order $p = 4$ if and only if

$${\omega_s}^2 \|\omega_s v_1^{dq} - \mathcal{J} v_2^{dq}\|^4 > 0 .$$

B. Differential observability of ω_r

We study now the observability of (3) from the knowledge of $u_s^{xy}, \imath_s^{xy}, \psi_s^{xy}$ and τ in (3d). The derivative $\frac{\mathrm{d}\tau}{\mathrm{d}t}$ determines uniquely $\omega_r \xi$ where we define

$$\xi := \langle \psi_s^{xy} - \sigma L_s \imath_s^{xy}, \psi_s^{xy} \rangle . \quad (5)$$

Then, differentiating once more, we get $\mathcal{D}_3 \begin{pmatrix} \omega_r \\ T_l \end{pmatrix}$ where $\mathcal{D}_3 := \begin{pmatrix} \xi & 0 \\ \frac{\mathrm{d}\xi}{\mathrm{d}t} & -\frac{n}{J}\xi \end{pmatrix}$. If ξ is non zero, $\mathcal{D}_3$ is invertible and (ω_r, T_l) is uniquely determined. More precisely, we get uniform strong differential observability if there exists $\ell > 0$ such that $\mathcal{D}_3(t)^\top \mathcal{D}_3(t) \geq \ell \mathcal{I}$ for all $t \geq 0$. Computing its trace and determinant, this is actually guaranteed if ξ and $\frac{\mathrm{d}\xi}{\mathrm{d}t}$ are bounded and $\xi(t)^2 > \ell'$ for some $\ell' > 0$ and for all $t \geq 0$. Pushing further the differentiation, using the fact that T_l is constant, the kth time derivative of the known signal $\omega_r \xi$ determines $\omega_r \xi^{(k)} + \alpha_k \tau \xi^{(k-1)}$ for some known constants α_k, so that similar results can be obtained if some derivative of ξ is uniformly positive. This is summed up next.

Lemma 3: The velocity subsystem (3) is differentially observable of order $p_v \geq 3$ if and only if the signal ξ defined in (5) is non zero or has one nonzero time derivative of order smaller than $p_v - 3$ at each time. Besides, if ξ and $\frac{\mathrm{d}\xi}{\mathrm{d}t}$ are bounded, it is uniformly strongly differentially observable of order 3 if there exists $\ell > 0$ such that $\xi(t)^2 > \ell$ for all $t \geq 0$.

Lemmas 1, 2 and 3 thus characterize the operating conditions providing differential observability and allowing to build a KKL observer. We recover in particular the observability singularities $\omega_s = 0$ and $\psi_s^{xy} = 0$ at steady state.

IV. The flux observer

Under the conditions exhibited in Section III-A ensuring the differential observability of the flux subsystem (2) with state $x_f := (\psi_s^{xy}, \jmath_s)$ at some order $p \geq 4$, a nonlinear Luenberger observer or KKL observer can be designed according to [9]. Considering a compact set $\mathcal{X}_f$ where solutions to (2) are known to evolve, the idea is to look for a mapping $(t, x_f) \mapsto \mathcal{T}(t, x_f)$ such that

- $t \mapsto \mathcal{T}(t, x_f(t))$ is solution to

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \Lambda z + \Gamma\big(\jmath_s - \langle \imath_s^{xy}, \psi_s^{xy} - \tfrac{\sigma L_s}{2}\imath_s^{xy}\rangle\big), \quad (6)$$

  along solutions $t \mapsto x_f(t)$ of (2) in $\mathcal{X}_f$ for some controllable pair (Λ, Γ) with Λ Hurwitz;
- $x_f \mapsto \mathcal{T}(t, x_f)$ becomes injective on $\mathcal{X}_f$ at least after a certain time.

Knowing that the output (2c) used in (6) is zero along solutions, $\mathcal{T}(t, x_f(t))$ is then known to follow the dynamics

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \Lambda z \quad (7)$$

which converges exponentially fast to zero and an estimate $\hat{x}_f(t)$ of $x_f(t)$ can thus be recovered by solving online

$$\mathcal{T}(t, \hat{x}_f(t)) = 0 , \qquad \hat{x}_f(t) \in \mathcal{X}_f , \quad (8)$$

at least after a certain time, exploiting the injectivity property of $\mathcal{T}$. The existence of a map immersing the dynamics into (6) is guaranteed according to [9, Theorem 1] for any choice of (Λ, Γ), and an explicit expression of such a map is provided in the next section. Then, we exploit [9, Theorem 2] to prove its injectivity for (Λ, Γ) of dimension p with sufficiently fast eigenvalues, where p is the order of uniform strong differential observability provided by Lemma 1.

A. *Design of $\mathcal{T}$ immersing* (2) *into* (6)

Consider $\Lambda \in \mathbb{R}^{p\times p}$ Hurwitz and $\Gamma \in \mathbb{R}^p$. The only nonlinearity of (2) comes from the quadratic terms $|\psi_s^{xy}|^2$. But noticing from (2) that ψ_s^{xy} depends linearly on its initial condition and that $\jmath_s$ is linear in its initial condition and quadratic in the one of ψ_s^{xy}, we conclude that the same holds for the output (2c), and thus for z in (6). This leads us to consider a candidate for $\mathcal{T}$ as

$$\mathcal{T}(\psi_s^{xy}, \jmath_s, t) := m(t)|\psi_s^{xy}|^2 + P^{xy}(t)\psi_s^{xy} + r(t)\jmath_s + s(t) \quad (9)$$

with time signals $m, r, s \in \mathbb{R}^p$ and $P^{xy} \in \mathbb{R}^{p\times 2}$ to be chosen so that $t \mapsto \mathcal{T}(\psi_s^{xy}(t), \jmath_s(t), t)$ evolves according to (6). Computing $\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{T}(\psi_s^{xy}(t), \jmath_s(t), t)$, replacing in (6), and identifying the terms in $|\psi_s^{xy}|^2, \psi_s^{xy}, \jmath_s$, we readily obtain

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \Lambda m - \frac{R_r}{\sigma L_s L_r} r \quad (10a)$$
$$\frac{\mathrm{d}P^{xy}}{\mathrm{d}t} = \Lambda P^{xy} + \omega_e P^{xy}\mathcal{J} - \Gamma(\imath_s^{xy})^\top - 2m(u_s^{xy} - R_s\imath_s^{xy})^\top - \frac{1}{\sigma L_s} r(u_s^{xy})^\top \quad (10b)$$
$$\frac{\mathrm{d}r}{\mathrm{d}t} = \Lambda r + \frac{R_+}{\sigma L_s} r + \Gamma \quad (10c)$$
$$\frac{\mathrm{d}s}{\mathrm{d}t} = \Lambda s + \frac{\sigma L_s}{2}|\imath_s^{xy}|^2\Gamma - P^{xy}(u_s^{xy} - R_s\imath_s^{xy}) + \frac{R_-}{2}|\imath_s^{xy}|^2 r \quad (10d)$$

The time dependence of $\mathcal{T}$ thus takes the form of filters of the known signals u_s^{xy}, $\imath_s^{xy}$ and ω_e, which can be implemented in real time from any initial conditions and for any choice of (Λ, Γ).

Note though that the filters are unstable if $\Lambda + \frac{R_+}{\sigma L_s}\mathcal{I}_p$ is not Hurwitz, namely if the chosen eigenvalues in Λ are too small. This is not a problem in theory, because any solution to the filters (even unbounded) ensures that $\mathcal{T}(t, x_f(t))$ converges exponentially to zero and an estimate can be obtained by solving (8) if $\mathcal{T}$ is injective with respect to x_f. However, handling unbounded signals in practice is undesirable. Ways around this problem include picking the eigenvalues of Λ sufficiently negative, or observing that m and r actually have time-invariant dynamics and can thus be chosen constant at their steady-state values. However, restricting the choice of Λ to a constant Hurwitz matrix with sufficiently negative eigenvalues may result in a limitation of performance, in particular in terms of robustness to noise. An alternative trick is to notice that the undesirable unstable term actually comes from $\frac{R_+}{\sigma L_s} r\jmath_s$, which equals $\frac{R_+}{\sigma L_s} r\langle \imath_s^{xy}, \psi_s^{xy} - \frac{\sigma L_s}{2}\imath_s^{xy}\rangle$ according to (2c). This alternative representation instead leads to the filter dynamics

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \Lambda m - \frac{R_r}{\sigma L_s L_r} r \quad (11a)$$
$$\frac{\mathrm{d}P^{xy}}{\mathrm{d}t} = \Lambda P^{xy} + \omega_e P^{xy}\mathcal{J} - \Gamma(\imath_s^{xy})^\top - 2m(u_s^{xy} - R_s\imath_s^{xy})^\top - \frac{1}{\sigma L_s} r(u_s^{xy} - R_+\imath_s^{xy})^\top \quad (11b)$$
$$\frac{\mathrm{d}r}{\mathrm{d}t} = \Lambda r + \Gamma \quad (11c)$$
$$\frac{\mathrm{d}s}{\mathrm{d}t} = \Lambda s + \frac{\sigma L_s}{2}|\imath_s^{xy}|^2\Gamma - P^{xy}(u_s^{xy} - R_s\imath_s^{xy}) - R_r\frac{L_s}{L_r}|\imath_s^{xy}|^2 r \quad (11d)$$

which are this time stable for any choice of Λ Hurwitz. Note that r and m can also be chosen constant at their steady state

$$r = -\Lambda^{-1}\Gamma \quad , \quad m = -\frac{R_r}{\sigma L_s L_r}\Lambda^{-2}\Gamma. \quad (12)$$

B. *Inversion of $\mathcal{T}$*

When solving (8) for the map $\mathcal{T}$ defined in (9) with $p \geq 4$, it is tempting to consider $|\psi_s^{xy}|^2$ as an independent variable ϕ and solve the linear system

$$\begin{pmatrix} m(t) & P^{xy}(t) & r(t) \end{pmatrix} \begin{pmatrix} \widehat{\phi}(t) \\ \widehat{\psi_s^{xy}}(t) \\ \widehat{\jmath_s}(t) \end{pmatrix} = -s(t). \quad (13)$$

However, the full rank of this matrix is in general not guaranteed by the observability of (2) since it does not use the fact that $\phi = |\psi_s^{xy}|^2$. The following theorem shows it can be done nonetheless.

Theorem 1: Consider a compact set $U \subset \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}$ and a positive scalar $\ell_H > 0$. Then, for any input $(\imath_s^{xy}, u_s^{xy}, \omega_e)$ with values in U making the flux subsystem (2) uniformly strongly differentially observable of order $p \geq 4$ with parameter $\ell_H > 0$, and for any controllable pair (Λ_0, Γ) with $\Lambda_0 \in \mathbb{R}^{p\times p}$ Hurwitz and $\Gamma \in \mathbb{R}^p$, there exists $k^* > 0$ and $q^* > 0$ such that for all $k > k^*$, there exists $t_k > 0$ such that for any solution to (11),

$$\begin{pmatrix} m(t) & P^{xy}(t) & r(t) \end{pmatrix}^\top \begin{pmatrix} m(t) & P^{xy}(t) & r(t) \end{pmatrix} \geq \frac{q^*}{k^p}\mathcal{I}_4 \quad \forall t > t_k \ , \quad (14)$$

with m, P^{xy}, r obtained from (10) for $\Lambda = k\Lambda_0$.

It follows from Theorem 1 that for all $t > t_k$, (13) admits a unique solution and thanks to the uniformity given by (14),

$$\lim_{t\to\infty} |\widehat{\psi_s^{xy}}(t) - \psi_s^{xy}(t)| + |\widehat{\jmath_s}(t) - \jmath_s(t)| = 0 \ . \quad (15)$$

The lower-bound q^* is directly related to the observability parameter ℓ_H, while the time t_k is mostly related to the decay rate given by Λ and tends to 0 when k is pushed to infinity. Notice though that the conditioning of the inversion in (13) degrades as k goes to infinity, so that pushing the eigenvalues of Λ too far is generally not a good idea. Therefore, this *high gain* result provides existence of the KKL observer but the choice of the pair (Λ, Γ) needs to be optimized and adapted to the system dynamics to increase performance and robustness as will be seen in Section VI. In particular, it can be checked that the matrix $\begin{pmatrix} m & P^{xy} & r \end{pmatrix}$ is invertible at steady state for any choice of Λ Hurwitz under the conditions of Lemma 2.

Proof: The proof relies on observing that subsystem (2) can be immersed into the time-varying state-affine system

$$\begin{aligned} \frac{\mathrm{d}\psi_s^{xy}}{\mathrm{d}t} &= u_s^{xy} - R_s \imath_s^{xy} - \omega_e \mathcal{J} \psi_s^{xy} \\ \frac{\mathrm{d}\phi}{\mathrm{d}t} &= 2\langle \psi_s^{xy}, u_s^{xy} - R_s \imath_s^{xy} \rangle \\ \sigma L_s \frac{\mathrm{d}\jmath_s}{\mathrm{d}t} &= \tfrac{R_r}{L_r}\phi - R_+ \jmath_s + \langle \psi_s^{xy}, u_s^{xy} \rangle - \tfrac{\sigma L_s}{2} R_- |\imath_s^{xy}|^2 \\ 0 &= \jmath_s - \langle \imath_s^{xy}, \psi_s^{xy} - \tfrac{\sigma L_s}{2} \imath_s^{xy} \rangle \end{aligned}$$

obtained by considering $\phi := |\psi_s^{xy}|^2$ and ψ_s^{xy} as independent variables. It turns out that reproducing the reasoning of Section III-A on this new extended system gives the exact same condition for uniform strong differential observability. In other words, (2) is uniformly strongly differentially observable of order $p \geq 4$, if and only if the system with extended state $(\psi_s^{xy}, \phi, \jmath_s)$ is. Applying [9, Theorem 2] to the extended system gives the result for $\mathcal{S} = \mathbb{R}^2 \times \mathbb{R} \times \mathbb{R}$ exploiting the linearity of the maps. ■

Remark 1: The proof relies on immersing the subsystem (2) into a linear system by considering $|\psi_s^{xy}|^2$ as an independent state. From that point other linear observers could have been used. For instance, we consider a Kalman filter in [8].

V. The velocity observer

We now assume that we know an estimate $\widehat{\psi_s^{xy}}$ of ψ_s^{xy} besides $\imath_s^{xy}, u_s^{xy}, \omega_e$, and we consider the velocity subsystem (3), which is state-affine with injection of $(u_s^{xy}, \imath_s^{xy}, \psi_s^{xy})$. Due to linearity and under an assumption of uniform differential observability [12], a first possibility is to design a Kalman observer. This route is followed in [8].

Here instead, we follow the spirit of this paper by designing a Luenberger observer. Because system (3) can be seen as a time-varying linear system with known input $\langle \psi_s^{xy} - \sigma L_s \imath_s^{xy}, \mathcal{J} u_s^{xy} \rangle$ and output τ, we look for a time-varying linear map

$$\mathcal{T}_v(t, \omega_r, \tau, T_l) = a(t)\omega_r + b(t)\tau + c(t)T_l \tag{16}$$

transforming (3) into the contracting dynamics

$$\frac{\mathrm{d}z}{\mathrm{d}t} = Az + B\tau + B_u \langle \psi_s^{xy} - \sigma L_s \imath_s^{xy}, \mathcal{J} u_s^{xy} \rangle \tag{17}$$

where $\tau = \langle \imath_s^{xy}, \mathcal{J}\psi_s^{xy} \rangle$ is known, $A \in \mathbb{R}^{p_v \times p_v}$ Hurwitz and $B \in \mathbb{R}^{p_v}$ are design parameters such that the pair (A, B) is controllable, $B_u \in \mathbb{R}^{p_v}$ is an appropriate matrix and the dynamics of $a, b, c \in \mathbb{R}^{p_v}$ are to be defined.

Differentiating $\mathcal{T}_v$ in time and replacing in (17) yields

$$\frac{\mathrm{d}a}{\mathrm{d}t} = Aa + \frac{1}{\sigma L_s} \langle \psi_s^{xy} - \sigma L_s \imath_s^{xy}, \psi_s^{xy} \rangle b \tag{18a}$$

$$\frac{\mathrm{d}b}{\mathrm{d}t} = Ab + \frac{R_t}{\sigma L_s} b + B - \frac{n^2}{J} a \tag{18b}$$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = Ac + \frac{n}{J} a \tag{18c}$$

and $B_u = -\frac{1}{\sigma L_s} b$, namely

$$\frac{\mathrm{d}z}{\mathrm{d}t} = Az + B\langle \imath_s^{xy}, \mathcal{J}\psi_s^{xy} \rangle - \frac{1}{\sigma L_s} b \langle \psi_s^{xy} - \sigma L_s \imath_s^{xy}, \mathcal{J} u_s^{xy} \rangle. \tag{18d}$$

A drawback of those filters is that the destabilizing term $\frac{R_t}{\sigma L_s} b$ requires the eigenvalues of A to be larger than $\frac{R_t}{\sigma L_s}$ and that the dynamics of a and b are interconnected leading to an additional constraint on A whose eigenvalues need to be sufficiently large. As above, thoses issues are not problematic theoretically, but reduce the scope of possible choices of A if stable filters are desired. This in turn may limit the possible performance, in particular in terms of robustness to noise. As above, a trick to avoid those problems is to notice that those undesirable terms are created proportionally to the output τ, known according to (3d). It thus leads to an alternative definition of cascaded filters ($b \to a \to c, z$)

$$\frac{\mathrm{d}a}{\mathrm{d}t} = Aa + \frac{1}{\sigma L_s} \langle \psi_s^{xy} - \sigma L_s \imath_s^{xy}, \psi_s^{xy} \rangle b \tag{19a}$$

$$\frac{\mathrm{d}b}{\mathrm{d}t} = Ab + B \tag{19b}$$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = Ac + \frac{n}{J} a \tag{19c}$$

$$\frac{\mathrm{d}z}{\mathrm{d}t} = Az - \frac{1}{\sigma L_s} \langle \psi_s^{xy} - \sigma L_s \imath_s^{xy}, \mathcal{J} u_s^{xy} \rangle b \tag{19d}$$

$$+ \langle \imath_s^{xy}, \mathcal{J}\psi_s^{xy} \rangle \left(B - \frac{R_t}{\sigma L_s} b + \frac{n^2}{J} a \right) \tag{19e}$$

which are thus stable if A is Hurwitz. An additional advantage is that b can be chosen constant equal to $A^{-1}B$.

Finally, estimates are obtained by solving online the linear equation

$$\begin{pmatrix} a(t) & b(t) & c(t) \end{pmatrix} \begin{pmatrix} \hat{\omega}_r \\ \hat{\tau} \\ \hat{T}_l \end{pmatrix} = z(t). \tag{20}$$

Similarly to Theorem 1, the differential observability of order $p_v \geq 3$ given by Lemma 3 allows to obtain from [9, Theorem 2] the uniform full rank of the matrix $\begin{pmatrix} a(t) & b(t) & c(t) \end{pmatrix}$ after a certain time, for some controllable pairs (A, B) of dimension p_v, and thus asymptotic convergence of the estimates $(\hat{\omega}_r, \hat{\tau}, \hat{T}_l)$ when ψ_s^{xy} is exactly known.

When replacing ψ_s^{xy} by its asymptotic estimate $\widehat{\psi_s^{xy}}$ in (19), we obtain estimates $\hat{a}, \hat{b}, \hat{c}, \hat{z}$ converging asymptotically to (a, b, c, z) thanks to the stability of the filters. Besides, the *input-to-state stability* (ISS) of stable filters ensures that persistent errors in $\widehat{\psi_s^{xy}}$ – for instance due to noise – or modelling errors – for instance due to varying torque – lead to stable practical estimates $\hat{a}, \hat{b}, \hat{c}, \hat{z}$, and thus degraded estimates in (20). Note that large disturbances leading to large errors in (20) do not have any impact on the stability of the filters and the future estimates since (20) is decoupled from (19). Besides, the gain between those disturbances and $(\hat{\omega}_r, \hat{T}_l)$ can be minimized at the first order by optimizing the associated transfer function via a proper choice of the pair (A, B) as suggested in the next section.

VI. Steady state analysis and optimization of observer eigenvalues

All in all, our observer for system (1) consists in

- implementing (11) from any initial condition and solving at each time (13);
- implementing (19) from any initial condition, with ψ_s^{xy} replaced by $\widehat{\psi_s^{xy}}$ and solving at each time (20).

for some controllable pairs (Λ, Γ) and (A, B) to be chosen. From a theoretical point of view, any such pair providing uniqueness of solutions in (13),(20) is eligible. However, in practice, other performance, such as the robustness to noise, are of crucial importance and a *good* choice of the observer parameters is naturally decisive.

The approach we propose to optimize this choice is to study the transfer functions characterizing the local behavior of the observers around their equilibrium in steady state when $\imath_s^{dq}$, u_s^{dq} and ω_s are constant. We therefore consider the dynamics of the filters in the dq frame.

Because we are essentially interested by the estimation of ω_r and T_l which uses the estimate $\widehat{\psi_s^{xy}}$ as input, we optimize the transfer from $(u_s^{xy}, \imath_s^{xy})$ to $\widehat{\psi_s^{xy}}$ in the flux observer, and the transfer from $(u_s^{xy}, \imath_s^{xy}, \widehat{\psi_s^{xy}})$ to ω_r and T_l in the velocity observer. Only the optimization of (Λ, Γ) is reported here due to space constraints.

Denoting $p^d \in \mathbb{R}^m$ and $p^q \in \mathbb{R}^m$ such that $P^{dq} = \begin{pmatrix} p^d & p^q \end{pmatrix}$, (11b) can be rewritten as

$$\dot{p}^d = \Lambda p^d + \omega_s p^q + \alpha_\Lambda u_s^d + \beta_\Lambda \imath_s^d \tag{21a}$$

$$\dot{p}^q = \Lambda p^q - \omega_s p^d + \alpha_\Lambda u_s^q + \beta_\Lambda \imath_s^q \tag{21b}$$

$$\dot{s} = \Lambda s - (u_s^d - R_s \imath_s^d) p^d - (u_s^q - R_s \imath_s^q) p^q + \gamma_\Lambda |\imath_s^{dq}|^2 \tag{21c}$$

with inputs $(u_s^d, u_s^q, \imath_s^d, \imath_s^q)$, where α_Λ, β_Λ and γ_Λ are vectors in $\mathbb{R}^m$ defined by

$$\alpha_\Lambda = -2m - \frac{1}{\sigma L_s} r \quad , \quad \beta_\Lambda = 2R_s m + \frac{R_+}{\sigma L_s} r - \Gamma$$

$$\gamma_\Lambda = \frac{\sigma L_s}{2}\Gamma - R_r \frac{L_s}{L_r} r$$

with m and r taken at their steady state values (12).

When ω_s, $\imath_s^{dq}$, u_s^{dq} are constant equal to $\bar{\omega}_s, \bar{u}_s^{dq}, \bar{\imath}_s^{dq}$ in the dq frame, the observer steady state is characterized by

$$\begin{pmatrix} \bar{p}^d \\ \bar{p}^q \\ \bar{s} \end{pmatrix} = -\bar{\Lambda}^{-1} \begin{pmatrix} \alpha_\Lambda \bar{u}_s^d + \beta_\Lambda \bar{\imath}_s^d \\ \alpha_\Lambda \bar{u}_s^q + \beta_\Lambda \bar{\imath}_s^q \\ \gamma_\Lambda |\bar{\imath}_s|^2 \end{pmatrix}$$

where

$$\bar{\Lambda} = \begin{pmatrix} \Lambda & \bar{\omega}_s \mathcal{I}_p & 0 \\ -\bar{\omega}_s \mathcal{I}_p & \Lambda & 0 \\ -(\bar{u}_s^d - R_s \bar{\imath}_s^d)\mathcal{I}_p & -(\bar{u}_s^q - R_s \bar{\imath}_s^q)\mathcal{I}_p & \Lambda \end{pmatrix}$$

and following Remark 1, the flux estimate given by

$$\bar{\psi}_s^{dq} = -\left[M_\Lambda \bar{P}^{dq}\right]^{-1} M_\Lambda \bar{s}$$

when $p = 4$.

Assume the inputs are noisy, namely we feed the observer with $u_s^{dq} + \nu_u, \imath_s^{dq} + \nu_i$. This noise propagates to noises ν_p and ν_s on P and s through (21) and then to the estimate $\widehat{\psi}_s^{dq}$. At the first order, around the equilibrium, (21) writes

$$\frac{d}{dt}\begin{pmatrix} \Delta p^d \\ \Delta p^q \\ \Delta s \end{pmatrix} = \bar{\Lambda} \begin{pmatrix} \Delta p^d \\ \Delta p^q \\ \Delta s \end{pmatrix} + \bar{L} \begin{pmatrix} \Delta u_s^d \\ \Delta u_s^q \\ \Delta \imath_s^d \\ \Delta \imath_s^q \end{pmatrix} \quad , \quad \Delta \widehat{\psi}_s^{dq} = \bar{\Sigma} \begin{pmatrix} \Delta p^d \\ \Delta p^q \\ \Delta s \end{pmatrix}$$

where

$$\bar{L} = \begin{pmatrix} \alpha_\Lambda & 0 & \beta_\Lambda & 0 \\ 0 & \alpha_\Lambda & 0 & \beta_\Lambda \\ -\bar{p}^d & -\bar{p}^q & R_s \bar{p}^d + 2\gamma_\Lambda \bar{\imath}_s^d & R_s \bar{p}^q + 2\gamma_\Lambda \bar{\imath}_s^q \end{pmatrix}$$

$$\bar{\Sigma} = -\left[M_\Lambda \bar{P}^{dq}\right]^{-1} \begin{pmatrix} \bar{\psi}_s^d M_\Lambda & \bar{\psi}_s^q M_\Lambda & M_\Lambda \end{pmatrix}$$

where we have used the fact that at the first order $[M + \Delta M]^{-1} = M^{-1} - M^{-1}[\Delta M] M^{-1}$ and so

$$\begin{aligned} \Delta \widehat{\psi}_s^{dq} &= \left[M_\Lambda \bar{P}^{dq}\right]^{-1} \left[M_\Lambda \Delta P^{dq}\right] \left[M_\Lambda \bar{P}^{dq}\right]^{-1} M_\Lambda \bar{s} \\ &\qquad - \left[M_\Lambda \bar{P}^{dq}\right]^{-1} M_\Lambda \Delta s \\ &= -\left[M_\Lambda \bar{P}^{dq}\right]^{-1} \left[M_\Lambda \Delta P^{dq}\right] \bar{\psi}_s^{dq} - \left[M_\Lambda \bar{P}^{dq}\right]^{-1} M_\Lambda \Delta s \end{aligned}$$

We then study the impact of the choice of (Λ, Γ) on the L_2 norm of the transfer function

$$F(\sigma) = \bar{\Sigma}[\sigma \mathcal{I}_{2p+1} - \bar{\Lambda}]^{-1} \bar{L} \ . \tag{22}$$

For that, we consider the pair (Λ, Γ) in controllability form

$$\Lambda = \begin{pmatrix} 0 & 0 & \cdots & \cdots & -\upsilon_0 \\ 1 & 0 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -\upsilon_{p-1} \\ 0 & \cdots & 0 & 1 & -\upsilon_p \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix} \tag{23}$$

where (υ_i) are the coefficients of the desired characteristic polynomial. We pick $p = 4$, assuming the criterion of Lemma 2 holds.

We propose to fix[1] the structure of this polynomial to be that of a Bessel filter with a cut-off frequency ω_c to optimize. This allows to parametrize the pair by a single parameter and launch the criterion on a grid of ω_c. Figures 1 and 2 show the conditioning of the inversion at steady state and the L_2 norm of the transfer function for a certain range of ω_c around $\bar{\omega}_s$. It shows that indeed, the choice of ω_c has a dramatic impact on the performance of the observer. With the intuition that ω_c must imperatively be adapted to the system's time scale given by ω_s, we then plot on Figure 3

[1]One may enlarge the class of considered Λ by optimizing (υ_i) using a four-dimensional grid of real or complex conjugate eigenvalues. But this road did not provide a significant increase of performance in this particular example.

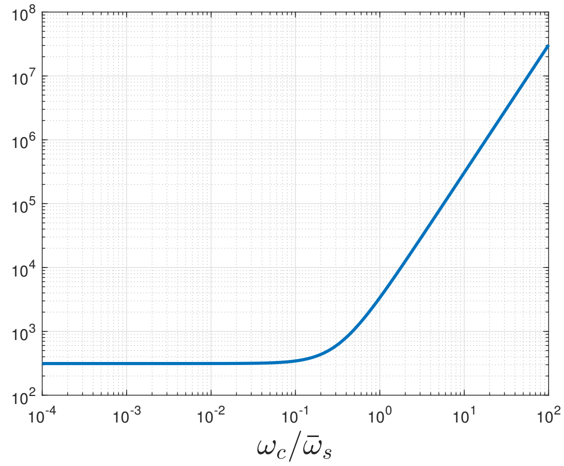

Fig. 1. Condition number of $M_\Lambda \bar{P}^{dq}$ as a function of the Bessel cut-off frequency at nominal parameters (log-scale)

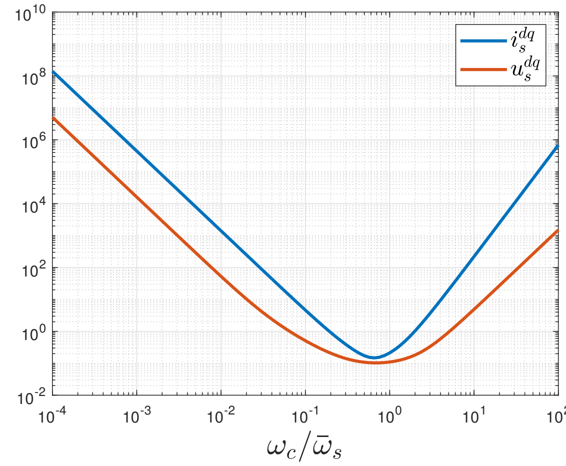

Fig. 2. L_2 norm of the transfer function (22) between $(\imath_s^{dq}, u_s^{dq})$ and $\widehat{\psi}_s^{dq}$ at nominal parameters (log-scale)

the value of the optimal ω_c as a function of $\bar{\omega}_s$: we find a linear dependance with slope 0.56, which suggests that ω_c should be dynamically adapted to ω_s. This is confirmed on simulations in Section VII. Actually, based on a dimensional analysis, we note that the system and observer dynamics are invariant under the following transformations parameterized by the scalar multipliers $(t_n, \ell_n, m_n, \imath_n, \gamma_n)$

$$(u, \imath, \psi, t, \omega, R, L, P) \rightarrow \left(\frac{t_n^3 \imath_n}{m_n \ell_n^2} u, \frac{1}{\imath_n} \imath, \frac{t_n^2 \imath_n}{m_n \ell_n^2} \psi, \frac{1}{t_n} t, t_n \omega, \frac{t_n^3 \imath_n^2}{m_n \ell_n^2} R, \frac{t_n^2 \imath_n^2}{m_n \ell_n^2} L \right)$$

$$(P, s, \Lambda, \Gamma) \rightarrow \left(\frac{t_n^3 \imath_n \gamma_n}{m_n \ell_n^2} P, \frac{t_n^7 \imath_n^2 \gamma_n}{m_n^2 \ell_n^4} s, t_n \Lambda, \gamma_n \Gamma \right)$$

Hence, when the resistances are small enough to have a negligible effect, provided the cost on the function $\omega \mapsto F(j\omega)$ does not introduce ω dependent weights (verified by any L^p-norm), we can optimize the transfer function for the particular case $\bar{\omega}_s = 1$. Then the optimum for other values of $\bar{\omega}_s$ is obtained by multiplying Λ by $\bar{\omega}_s$.

From this analysis, it is actually tempting to adapt the choice of Λ dynamically in function of ω_s, namely take

$$\Lambda(t) = \omega_s(t) \Lambda_0 \tag{24}$$

with Λ_0 optimized for $\bar{\omega}_s = 1$. This is rendered possible by the fact that $\mathcal{T}$ defined in (9) still transforms the dynamics into (6) and thus (7) when Λ is time-varying. In other words, as long as ω_s ensures (7) is exponentially stable, $\mathcal{T}(\psi_s^{xy}(t), \jmath_s(t), t)$ converges to zero for any solution to (11). Estimates are then recovered by solving (13) online if the injectivity provided by Theorem 1 is preserved. Simulations in Section VII show the great interest of using (24) in presence of noise.

VII. Simulation

We apply the observer to the 4 kW motor in [13]. For that, we implement in the $\alpha\beta$ frame the flux observer made of

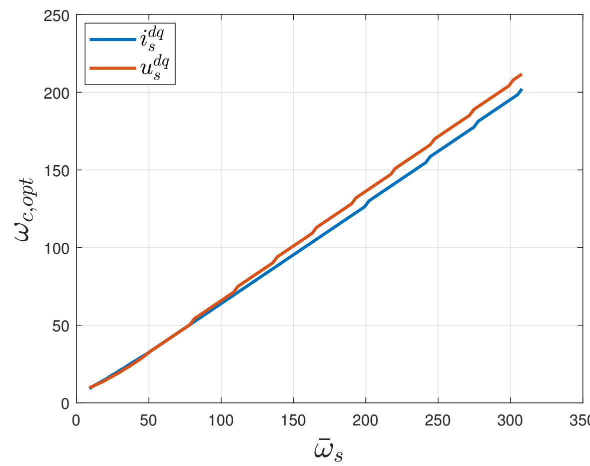

Fig. 3. Bessel cut-off frequency minimizing the L_2 norm of the transfer function between $(\imath_s^{dq}, u_s^{dq})$ and $\widehat{\psi}_s^{dq}$ as a function of $\bar{\omega}_s$

(11),(13) and the velocity observer made of (19),(20) with $\psi_s^{\alpha\beta}$ replaced by its estimate $\widehat{\psi_s^{\alpha\beta}}$ in (19) and for $p = 4$ and $p_v = 3$. The matrix Λ is chosen time-varying of the form (24), with (Λ_0, Γ) in controllability form (23) with (υ_i) given obtained from a Bessel filter with $\omega_{c,0} = 0.65\,\mathrm{rad\,s^{-1}}$. The pair (A, B) was taken in controllability form (23) with eigenvalues $(-20, -60, -300)$. The filters are all initialized at 0 and the resolution of (13),(20) launched only after a delay equal to 10 times the slowest real part of the eigenvalues in Λ and A. Indeed, this allows the filters to "forget" their initial conditions and avoid any bad conditioning at the initialization. The scenario of test along with the results are displayed in Fig. 5. In this scenario, the voltage input is assumed known (imposed by control), while the currents are measured with a band-limited white noise generated by Simulink with parameters `Noise Power`=$1 \times 10^{-5}\,\mathrm{A^2\,rad^{-1}\,s}$ and `Sample time`=1×10^{-3} s. The dynamic optimization of the choice of Λ presented in Section VI allows to successfully

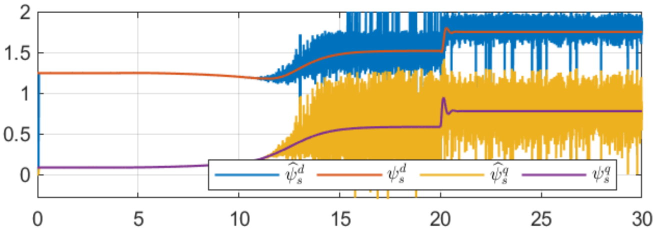

Fig. 4. Results in simulation of the flux observer (11),(13) for a constant Λ. Horizontal axis shows time and all signals are in SI units.

filter out the noise introduced by $\imath_s^{xy}$ in $\widehat{\psi_s^{xy}}$, while it is not optimized for $\widehat{J_s}$. For the sake of comparison, we show in Figure 4 the results for Λ constantly equal to $\frac{3}{4}\bar{\omega}_s\Lambda_0$: the noise gets more amplified as ω_s becomes smaller.

VIII. Conclusion

We propose a novel cascaded model of the induction motor, that is well-suited for observer design without time-scale separation. The KKL methodology appears well adapted for velocity and torque estimation and this application exhibits the dramatic importance of optimizing the KKL parameters for performance and robustness. This is done here around equilibrium points via linear-based arguments but a general theory of this tuning still needs to be developed.

References

[1] J. Holtz, "Sensorless control of induction machines - with or without signal injection?" *IEEE Transactions on Industrial Electronics*, vol. 53, no. 1, pp. 7–30, 2006.

[2] C. Lascu, I. Boldea, and F. Blaabjerg, "Comparative study of adaptive and inherently sensorless observers for variable-speed induction-motor drives," *IEEE Transactions on Industrial Electronics*, vol. 53, no. 1, pp. 57–65, 2006.

[3] J. Maes and J. A. Melkebeek, "Speed-sensorless direct torque control of induction motors using an adaptive flux observer," *IEEE Transactions on Industry Applications*, vol. 36, no. 3, pp. 778–785, 2000.

[4] Y.-R. Kim, S.-K. Sul, and M.-H. Park, "Speed sensorless vector control of induction motor using extended kalman filter," *IEEE Transactions on Industry Applications*, vol. 30, no. 5, pp. 1225–1233, 1994.

[5] M. Tursini, "Adaptive sliding-mode observer for speed-sensorless control of induction motors," *IEEE Transactions on Industry Applications*, vol. 36, no. 5, pp. 1380–1387, 2000.

[6] Z. Yan, C. Jin, and V. I. Utkin, "Sensorless sliding-mode control of induction motors," *IEEE Transactions on Industrial Electronics*, vol. 47, no. 6, pp. 1286–1297, 2000.

[7] P. Bernard, A. K. Jebai, and P. Martin, "Higher-order singular perturbations for control design with application to the control of induction motors," in *2020 59th IEEE Conference on Decision and Control (CDC)*, 2020, pp. 769–776.

[8] P. Bernard, T. Devos, A. K. Jebai, P. Martin, and L. Praly, "Sensorless position estimation of Permanent-Magnet Synchronous Motors using a nonlinear magnetic saturation model," in *International Conference on Electrical Machines, ICEM 2022*, 2022.

[9] P. Bernard, "Luenberger observers for nonlinear controlled systems," *IEEE Conference on Decision and Control*, 2017.

[10] P. Bernard and V. Andrieu, "Luenberger observers for non autonomous nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 64, no. 1, pp. 270–281, 2019.

[11] P. Krause, O. Wasynczuk, S. Sudhoff, and S. Pekarek, *Analysis of Electric Machinery and Drive Systems*. Wiley, 2013.

[12] P.-J. Bristeau, N. Petit, and L. Praly, "Design of a navigation filter by analysis of local observability," in *IEEE Conference on Decision and Control*, 2010, pp. 1298–1305.

[13] F. Jadot, F. Malrait, J. Moreno-Valenzuela, and R. Sepulchre, "Adaptive regulation of vector-controlled induction motors," *IEEE Transactions on Control Systems Technology*, vol. 17, no. 3, pp. 646–657, 2009.

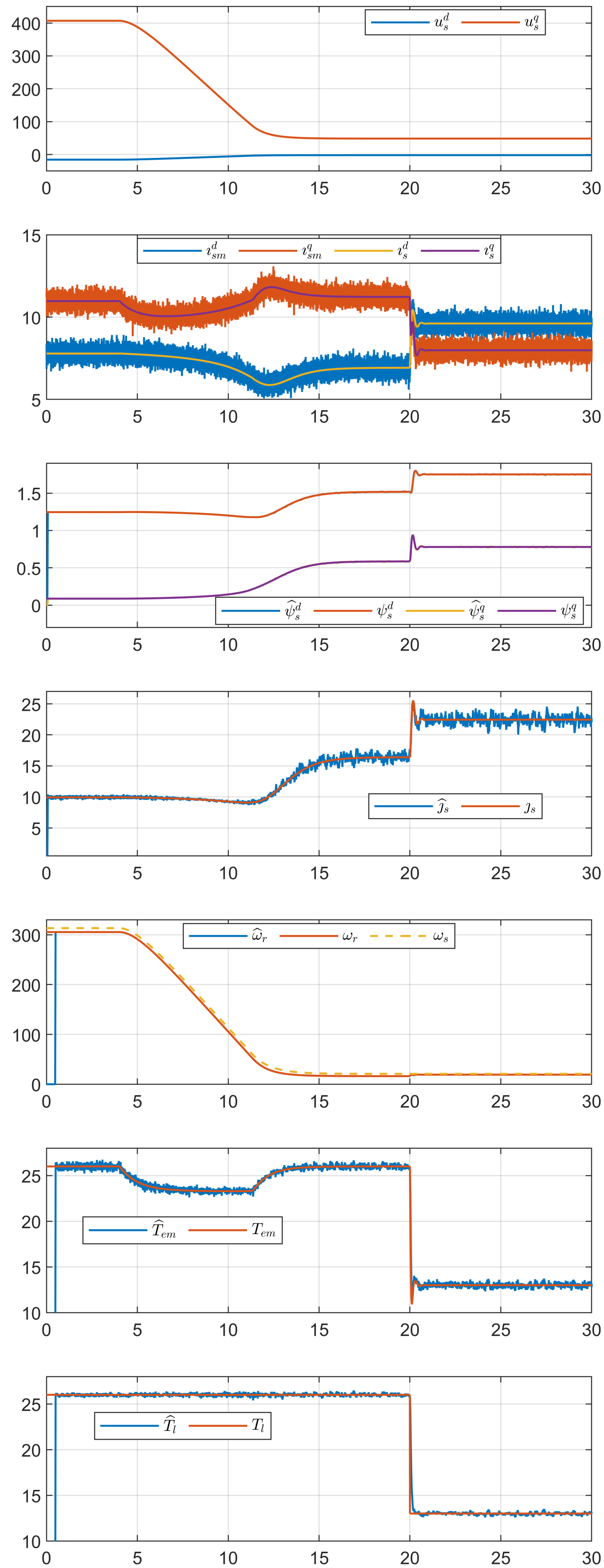

Fig. 5. Results in simulation of the flux observer (11),(13) and the velocity observer (19),(20). Horizontal axis shows time and all signals are in SI units.