ON PERFORMANCE IMPROVEMENT OF ADAPTIVE TUNERS*

Yu Tang[+], Romeo Ortega[+], Laurent Praly[‡]

Abstract: Here we depart from the standard stabilization objective of adaptive control and assume that for the plant to be controlled a parametrization of the regulator insuring closed-loop stability is **known**. The structure, dynamic order and number of adjustable parameters of the compensator are at the designer's disposal and no assumptions, except linearity are imposed on the plant. We propose an on-line tuning procedure for the controller parameters intended to improve performance and such that global stability is preserved. Performance of the adaptive system is evaluated in two ways: deriving **uniform bounds** on the sup value of the tracking error; giving conditions under which **RMS** performance index **decreases** when adaptation is turned on.

Note: *This is a shortened version of the original paper which is available upon request to the second author. All proofs are omited.*

I. INTRODUCTION

Adaptive control was originally motivated by the problem of on-line adjustment of some controller parameters to insure acceptable performance for linear plants with unknown or time varying coefficients. A landmark in the development of the field was the proof, circa 1980, that several adaptive control algorithms asymptotically attain the optimal performance for a relatively large family of plants. Performance was expressed in terms of output error tracking, usually referred to as model reference matching. The aforementioned family of plants $\{G_k(s)\}$ is continuously parameterized by a vector $k \in \mathbb{R}^m$. It was assumed the following hold: stable invertibility, known sign of the high frequency gain, relative degree and upperbound on the order of the system. Interestingly enough no assumption regarding the parameter vector is required, but instead it is allowed to **range on all** $\mathbb{R}^m$. Leaving aside the problem that model reference adaptive controllers are only concerned with tracking error, neglecting the disturbance rejection objectives, these controllers suffer from two main drawbacks: first, the usual procedures assume only that the plant is a black box of known order, therefore the requirement of model matching dictates the **controller structure**. Thus no advantage is taken of much information which is usually available about the plant, but in the estimator initialization; second, the **number of adjustable parameters** is determined by the assumed order of the plant and not by the number of adjustable parameters which the controller needs to achieve acceptable performance.

The proof that "mild" deviations of the plant from the class described above could cause the adaptive control algorithms to go unstable, triggered the interest on the 1980's on robustness issues. Most researchers focused their attention on modifying the parameter estimator and preserving the model reference controller structure in such a way to insure **boundedness** of the signals for plants that do not belong to the aforementioned class. That is, the main thrust of the research was focused on **stabilizing** a larger class of plants. This problem is of unquestionable theoretical interest and we refer the reader to [4,5] for an account of some of the existing results.

In several practical applications, designing a **fixed** parameter controller that stabilizes the plant can be accomplished in spite of the reduced prior information, see e.g. [6]. Tuning some of the controller parameters in a neighborhood of the stabilizing set is then required to improve the performance. It is clear that the tuning procedure should at least guarantee that overall global stability is preserved. To make the procedure of practical interest the conditions for stability should not rely on assumptions on the plant like known order or stable invertibility. Also it is desirable to avoid restrictions on the choice of the controller structure.

In this paper we **assume** knowledge of a regulator parametrization that stabilizes a family of plants and propose an on-line tuning procedure for the controller parameters intended to improve performance. It is only required the plant to be linear time invariant (LTI) and the controller to be linear in the adjustable parameters. Performance is evaluated in terms of output tracking error capabilities. In spite of the fact that the controller parameters are continuously updated, we use the term **tuning** to refer to the proposed control design. This to highlight the fact that we are applying an adaptive controller to a plant for which we know a controller parametrization that insures closed-loop stability. The task of the estimator is then to search on the neighborhood of the stabilizing set for a controller which yields "smaller" tracking errors. That is, our objective is, instead of stabilizing a possibly unstable plant, to **improve performance** via on-line parameter tuning.

Instrumental for our analysis is the introduction of a new estimator which includes a

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[+] National University of Mexico, P. O. Box 70-256, 04510 Mexico D.F., MEXICO.

[‡] CAI Ecole des Mines, 35 Rue Saint Honore, 77305 Fontainebleau, France.

signal normalizacion [1] and time varying σ-modification similar to those reported in [2,3]. The main contributions of the paper are as follows: 1) A new parameter estimator that yields an error model, suitable for the stability analysis of adaptive tuning problems is derived. This error model is described by a set of differential equations such that for all possible solutions they can be treated as linear time varying with all parameters **uniformly bounded.** 2) Conditions for **stability** of the overall system are given in terms of the stability margin of the plant in closed loop with the fixed stabilizing controller and the adaptation speed. 3) **Performance** of the adaptive system is evaluated in two different ways: deriving a uniform bound on the sup value of the tracking error, which depends on the sup value of the error obtained with the fixed stabilizing controller; giving conditions under which the performance index decreases when adaptation is turned on. Conditions that insure the unperturbed error equations to be exponentially stable are also derived.

The remainder of the paper is organized as follows: we present in Section 2 the problem formulation and the proposed estimator. The new error model, obtained via a state variable transformation, is derived in Section 3. In Section 4 we carry out the stability analysis of the error model. Conditions for L_∞ and exponential stability are derived and a global stability/instability boundary [10] is established. In Section 5 we present the analysis for performance improvement. Finally, we end up the paper with some concluding remarks.

II. PROBLEM STATEMENT AND PROPOSED SCHEME

The plant to be adaptively tuned is represented by

$$y(t)=G(p)u(t) \tag{2.1}$$

where $p:=d/dt$, $G(p)\in\mathcal{R}(p)$ is an n-th order strictly proper transfer function and $u(t)$ and $y(t)$ are the process input and output, respectively. Bounded external disturbances may also be considered in the model without affecting the results. They are omitted here for brevity.

Along the paper we will pursue as the design objective to impose a desired behavior to be plant output. We express this objective in terms of an output **tracking error** $e(t)$ and use the following criteria to evaluate performance

$$\operatorname{avg}[e^2(t)]:=\lim_{T\to\infty}\frac{1}{T}\int_{t_o}^{t_o+T} e^2(\tau)d\tau \tag{2.2a}$$

$$\|e(t)\|_\infty:=\sup_{t\geq 0}|e(t)| \tag{2.2b}$$

The control signal is taken as

$$u(t)=\hat{\theta}(t)^T\phi(t) \tag{2.3a}$$

where $\phi(t)\in\mathcal{R}^m$ is an auxiliary vector obtained by filtering the plants input, output and external reference

$$\phi(t)=F_\phi(p)\begin{bmatrix}u(t)\\y(t)\\r(t)\end{bmatrix} \tag{2.3b}$$

where $F_\phi(p)\in\mathcal{R}^{m\times 3}(p)$ is required to be strcitly proper and stable, $\hat{\theta}(t)\in\mathcal{R}^m$ is the control parameter vector to be adjusted on line.

Remark 2.1. For ease of representation we have chosen a controller structure where all the parameters are adjusted on line. In the case where only a few parameters are to be adjusted we can split $u(t)$ in two terms and lump the part of the controller with fixed parameters with the plant. As will become clear later, this does not affect any of the results below.

We require the knowledge of a vector θ_o such that for $\hat{\theta}(t)=\theta_o$ the closed-loop system (2.1), (2.3) is stable. It is clear that some prior knowledge, as described for instance in [6], is thus required. To improve the performance (e.g. when the plant changes) we then design an estimator that will search in a neighborhood of θ_o in the parameter space for a better regulator parametrization.

We propose the following **update law**

$$\dot{\hat{\theta}}(t)=-\sigma(t)\hat{\theta}(t)-\gamma\frac{\phi(t)e(t)}{\rho(t)^2}+\sigma(t)\theta_o,\quad \hat{\theta}(0)=\theta_o,\quad \gamma>0$$

$$\dot{\rho}(t)=-\mu\rho(t)+f,\quad \rho(0)>0,\quad \mu>0$$

$$\sigma(t)=\sigma_o+\frac{\dot{\rho}(t)}{\rho(t)},\quad \sigma_o\geq 0 \tag{2.4}$$

where f is a function that depends on the choice of the controller structure(therefore on $\phi(t)$), and chosen such that

i) $\rho(t)>\varepsilon_1$ for some $\varepsilon_1>0$ (2.5a)

ii) $\dot{\rho}(t)\geq-\mu\rho(t)$ (2.5b)

iii) $\frac{1}{\rho(t)}\|\phi(t)\|:=\frac{1}{\rho(t)}\max_i|\phi_i(t)|\leq 1, \forall t\geq 0.$ (2.5c)

A general function f for the estimator (2.4) is given in Lemma A.1 in the Appendix.

Three are the key features of the proposed estimator:

1. It includes a normalization factor $\rho(t)$, which has been proven essential for all recent robustness studies [5]. The important point to note here is that $\rho(t)$ is needed to bound $\|\phi(t)\|$ and not the effect of the **unmodeled dynamics** as in the robust stabilization problem, see [14]. Therefore it does not require prior information on the plant for its implementation.
2. The estimator is driven by a term θ_o which represents the best *a priori* estimate for the controller. Notice that if the estimate is good, that is, $e(t)$ is small, then $\hat{\theta}(t)$ will remain close to θ_o. Otherwise it will depart from it at a speed essentially determined by γ.
3. A new σ-modification, heretofore referred to as ρ-modification, has been added. The motivation for this choice of $\sigma(t)$ may be explained as follows. Since f dependes on $\phi(t)$, roughly speaking we can say that $\dot{\rho}(t)/\rho(t)$ will be large when $\phi(t)$ has high frequency content. Viewing $\sigma(t)$ as a forgetting factor, we see that in that

case the estimates, which may be inadequate because of the excitation of the high frequency parasitics, are rapidly forgotten.

Remark 2.2. The proposed ρ-modification is closely related to the e_1-modification used in [2]. Notice that in the particular case of regulation (r(t=0) with a single output feedback controller ($\phi(t)=y(t)$) the ρ-modification and the e_1-modification with normalization essentially coincide.

Remark 2.3. To be able to state our stability theorems in terms of designer chosen parameters we need to **know** the upperbound of $\|\phi(t)/\rho(t)\|$, e.g., one. To attain this bound, the regressor vector should contain only filtered signals. This **explains** the need for $G_\phi(p)$ to be strictly proper. It is important to remark that no condition is imposed on the bandwidth of F(p), thus the effect of F(p) is **negligible** for all practical purposes. Further explanation are given in the appendix.

II. THE NEW ERROR MODEL

In this section we derive the new error model. In contrast with the standard procedure, for our adaptive tuning problem the error model gives the tracking error in terms of the deviation of the actual controller parameters $\hat{\theta}(t)$ with respect to the known stabilizing parameters θ_o. To this end define

$$\tilde{\theta}(t):=\hat{\theta}(t)-\theta_o. \tag{3.1a}$$

Writting the control law (2.3) in terms of $\tilde{\theta}(t)$, replacing in (2.1) and arranging terms we obtain the standard error equation (see e.g.[9]).

$$e(t)=H_o(p)\tilde{\theta}(t)^T\phi(t)+e_o(t) \tag{3.1b}$$

where $H_o(p)\in R(p)$ is the transfer function $u(t) \rightarrow y(t)$ that results when $\hat{\theta}(t)$ is frozen at θ_o, that is when

$$u_o(t):=\theta_o^T\phi(t) \tag{3.1c}$$

and $e_o(t)$ is the corresponding tracking error.

We need the following key assumption:

A.1 θ_o **stabilizes** the plant. That is there exists positive constants m_o, λ_o such that

$$|h_o(t)|\le m_o e^{-\lambda_o t}, \quad \forall t\ge 0. \tag{3.2}$$

To get the new error model we introduce the following variable

$$z(t):=\tilde{\theta}(t)\rho(t). \tag{3.3}$$

This change of variable is a key step in all subsequent analysis. It is easy to see from (2.4) that

$$z(t)=-H_1(p)\bar{\phi}(t)e(t) \tag{3.4a}$$

$$\bar{\phi}(t):=\phi(t)/\rho(t) \tag{3.4b}$$

$$H_1(p)\triangleq\frac{\gamma}{p+\sigma_o} \tag{3.4c}$$

We can write the error equation (3.1b) in terms of z(t) as

$$e(t)=H_o(p)z(t)^T\bar{\phi}(t)+e_o(t) \tag{3.5}$$

The new error model is given then by (3.4), (3.5). Its key feature is that it is described by a differential equation which for all possible solutions can be treated as linear time-varying with **uniformly bounded** coefficients, namely

$$\dot{\omega}(t)\triangleq\begin{vmatrix}\dot{z}(t)\\ \dot{x}(t)\end{vmatrix}=\begin{vmatrix}-\sigma_o I_m & -\lambda\bar{\phi}(t)c_o^T\\ b_o\bar{\phi}(t)^T & A_o\end{vmatrix}\omega(t)-\begin{vmatrix}\gamma\bar{\phi}(t)\\ 0\end{vmatrix}e_o \tag{3.6}$$

where $x(t)\in R^{n+m}$ is a state of $H_o(p)$ with realization

$$H_o(p)=c_o^T(pI-A_o)^{-1}b_o \tag{3.7}$$

and

$$e(t)=c_o^Tx(t)+e_o(t). \tag{3.8}$$

The error model **defines** a feedback system. Notice that, in view of (2.5c), (3.4b), the vector signal entering the multipliers is **uniformly bounded**.

Remark 3.1. It is interesting to note that the ρ-modified estimator is a pseudogradient descent for the criterion

$$J_\rho:=\frac{1}{2}[e(t)^2+\sigma_o\rho(t)^2\|\tilde{\theta}(t)\|_E^2]$$

where $\|\cdot\|_E$ denots the Euclidean norm.

IV. STABILITY ANALYSIS

The error model (3.4), (3.5) (or its equivalent state-space representation (3.6)) have been exhaustively studied in the adaptive control literature. Both cases, when $\bar{\phi}(t)$ is possibly unbounded and when it is bounded, have been considered. The early results concerning this equation relied on a strictly positive realness (SPR) assumption of $H_o(p)$. A major contribution is due to [10] where, for the case of periodic, bounded $\bar{\phi}(t)$, and $\sigma_o=0$, the SPR assumption is replaced by a so-called "average SPR" condition. For a summary of the results pertaining (3.4), (3.5) when $H_o(p)$ is not SPR, and $\bar{\phi}(t)$ is possibly unbounded, see [4,5].

For the adaptive **tuning** problem conditions for stability are given in [8] for an estimator with fixed σ-modification (i.e., $\sigma\in R_+$ in (2.4a)) and normalization. The analysis relies on the L_∞ small gain theorem. It is shown that there always exists sufficiently small γ/σ such that L_∞-stability is insured independently of $H_o(p)$ or $\phi(t)$. The upperbound on γ/σ is dependent on $\|e_o(t)\|_\infty$ which is clearly a **signal dependent** quantity.

The use of the ρ-modified estimator proposed in this paper allows us to establish the following general stability results. First, we derive an **explicit** upperbound on the adaptation gain, that solely depends on the stability margin of $H_o(p)$, such that the system is L_∞-stable.

Second, the signal dependent average SPR condition of [10] is given in a **global context**. Third, conditions to insure **exponential stability** are given.

A. L_∞-stability analysis

Direct application of the small-gain theorem to the system (3.4),(3.5) yields the following result.

Theorem 4.1. Consider the plant (2.1) in closed loop with the controller (2.3) where the parameters are updated with (2.4). Choose f in order for (2.5) to hold. Assume that for the chosen θ_o Assumption A.1 holds. Under these conditions if γ, σ_o are chosen such that

$$\gamma/\sigma_0<\lambda_0/m_0 \tag{4.1}$$

whith λ_o, m_o as in (3.2), then for all bounded references and initial conditions

$$e(t),\ \rho(t)\in L_\infty,\ \phi(t),\hat{\theta}(t)\in L_\infty^m. \tag{4.2}$$

Furthermore,

$$\|e(t)\|_\infty\le K_1\|e_o(t)\|_\infty, K_1\triangleq 1/(1-\frac{\lambda m_o}{\sigma_o\lambda_o}). \tag{4.3}$$

□□□

Remark 4.1. Theorem 4.1 states that L_∞-stability of the adaptive tuner is preserved if $\gamma/\sigma_o<\lambda_o/m_o$. The result should be interpreted as follows, λ_0/m_0 quantifies **the relative stability** of the system in closed-loop with the *a priori* known stabilizing controller, i.e., $\theta_o^T\phi(t)$. On the other hand, γ defines the adaptation speed, and σ_o may be viewed as the constant part of the forgetting factor $\sigma(t)$. Since no restriction is imposed on σ_o, it may be taken arbitrarily large to insure stability. However, it is expected that large values of σ_o will reduce the effect of adaptation, restricting the adjustable parameters to a small neighborhood of θ_o.

Remark 4.2. From the proof of Theorem 4.1 it is easy to see that the assumption of **stationarity** on the plant can be easily removed [13]. This leads to a linear time varying operator $H_o(p,t)$ and Assumption A.1 should be replaced by

$$\sup_{t\ge0}\int_o^t |h_o(t,\tau)|d\tau\le\frac{\lambda_o}{m_o} \tag{4.8}$$

where $h_o(t,\tau)$ is the impulse response of $H_o(p,t)$.

Remark 4.3. The **generality** of the result can hardly be overestimated. The only restriction imposed on the plant is linearity, and on the controller, that be stable and linear in the adjustable parameters. No assumptions on the plant order, stable invertibility or sign of the high frequency gain are imposed. Also, we do not prescribe any particular synthesis methodology for the controller and the number of adjustable parameters is at the designer's disposal.

Remark 4.4. (4.3) gives a bound for the sup value of the adaptive system tracking error in terms of the error for the known fixed controller. However, it does not provide us with information about performance improvement since $K_1>1$.

B. Stability-instability boundary

Following the arguments of [11] we have the slow adaptation result below for the adaptive tuner with the ρ-modified estimator.

Theorem 4.2. ([11], see also [9,10]). Consider the adaptive system analyzed in Theorem 4.1 with $h_o(t)$ as in (3.2). Assume that $\bar{\phi}(t)$ is almost periodic with generalized Fourier series

$$\bar{\phi}(t)\ \sum_{\omega\in\Omega}\alpha(\omega)e^{j\omega t}, \forall t\in R_+ \tag{4.9}$$

where $\Omega\subset R$ are the distinct Fourier exponents and $\{\alpha(\omega),\ \omega\in\Omega\}$ are the Fourier coefficients. Define the matrix

$$B(\omega):=\sum_{\omega\in\Omega}\alpha(\omega)\alpha^*(\omega)^T H_o(-j\omega). \tag{4.10}$$

If $Re\lambda_i\{B(\omega)\}\neq0$, then $\exists\gamma_*>0$ such that $\forall\lambda\in(0,\gamma_*)$, system (3.6) is

i) exponentially stable if

$$Re\lambda_i\{B(\omega)\}>-\sigma_0/\gamma \tag{4.11}$$

ii) unstable if

$$\max_i Re\lambda_i\{B(\omega)\}<-\sigma_0/\gamma \tag{4.12}$$

□□□

Remark 4.5. An open issue that remains to be solved is how do we insure the existence of solutions to the adaptive system that will yield almost periodic $\bar{\phi}(t)$, see (4.9). Similar stability/instability results for the more general case when $\bar{\phi}(t)$ does not have a uniform average can be derived using the idea of sample average as in [11].

Remark 4.6. Applying the arguments of 3.5.3 in [9] to the new error model it is straightforward to see that the new signal dependent "average SPR" condition becomes

$$\sum_{\omega\in R} Re\{H_o(j\omega)\}Re\{\alpha(\omega)\alpha^*(\omega)\}>-\sigma_o/\gamma. \tag{4.13}$$

Thus, the condition imposed on the Nyquist locus of $H_o(p)$ is less restrictive and depends on the design parameters σ_o,γ. See (5.23) in [9]. Notice that, similarly to Theorem 4.1, robust stability is enhanced with small λ/σ_o.

C. Exponential stability

In this paragraph we give conditions under which the trivial equilibrium of the homogeneous part of the error equation (3.6) is exponentially stable.

Theorem 4.3. Consider the error equations (3.6)-(3.8). Assume $H_o(p)$ to be stable with no repeated poles and denote

$$\eta_j:=Re\{\lambda_j(A_0)\},\ j=1,2,\dots,m+n. \tag{4.14}$$

Let

$$\bar{\eta}:=\frac{1}{\min_j\{\eta_j\}}. \qquad (4.15)$$

Assume

$$1\geq\frac{m}{2}\bar{\eta}\|b_o\|_E+\frac{\gamma}{\sigma_o\sqrt{n+m}}+\frac{1}{2}\max\left\{\frac{2}{\sigma_o},\bar{\eta}\right\}(1+\varepsilon) \qquad (4.16)$$

holds for some $\varepsilon>0$. Under these conditions, $\exists P=P^T>0$ such that the Lyapunov function

$$V=\omega(t)^T P\omega(t) \qquad (4.17)$$

evaluated along the trajectories of (3.6), satisfies

$$\dot{V}\leq-\varepsilon V+c_1 e_o^2(t) \qquad (4.18a)$$

$$c_1\overset{\Delta}{=}\frac{\gamma^2}{2}\max\left\{\frac{2}{\sigma_o},\bar{\eta}\right\}. \qquad (4.18b)$$

Therefore, the unperturbed part of the system (3.6) is exponentially stable. Furthermore, the state vector of the closed loop plant $x(t)\in R^{n+m}$ belongs to the residual set

$$S:=\{x(t):\text{avg}\{\|x(t)\|_E^2\}\leq k,\ \forall t_o\geq 0, T>0\} \qquad (4.18c)$$

$$k:=\frac{2c_1}{\varepsilon}\max_i\{\eta_i\}\|e_o^2(t)\|_\infty. \qquad (4.18d)$$

Remark 4.7. As discussed in [9], the exponential stability of the unperturbed system equations brings out, via a total stability argument, important robustnes properties to be adaptive scheme. In particular, the bursting phenomena detected in [15] are **avoided.**

Remark 4.8. Condition (4.16) of the theorem requires the fixed controller to place the closed-loop poles sufficiently **far to the left** in the complex plane. This condition is certainly stronger than the one required for L_∞-stability. Also slow adaptation and large values of σ_o are required. The assumption of distinct poles for $H_0(p)$ is made, without loss of generality, to simplify the **interpretation** of the results. Exponential stability of the map $e_0(t)\rightarrow x(t)$ can also be established [7] imposing additionally an upperbound on σ_o.

V. PERFORMANCE IMPROVEMENT

In this section we are concerned with the problem of insuring that the use of the adaptive tuner improves the performance with respect to the performance attained with the fixed controller. To this end we consider the tracking error average performance index (2.2a)

$$J(\gamma):=\text{avg}[e^2(t)] \qquad (5.1)$$

where for convenience the dependence of J on the adaptation gain is explicitly shown. When $\gamma=0$, i.e., no adaptation, the plant is in closed-loop with the LTI controller (2.3b), (3.1c). The corresponding error and regressor signals are $e_o(t)$, $\phi_o(t)$, respectively. Thus the performance of the fixed controller is

$$J_\gamma(0)=\text{avg}[e_0^2(t)]. \qquad (5.2)$$

One way to determine if performance will be improved by the adaptive controller is to evaluate the performance-index sensitivity with respect to variations of the adaptation gain [16]. If we can show that, around $\gamma=0$, the sensitivity is negative, i.e.,

$$J_\gamma\overset{\Delta}{=}\frac{\partial J}{\partial\gamma}\Big|_{\gamma=0}<0 \qquad (5.3)$$

then it would imply that performance index **decreases** when the adaptation is turned on. The performance index sensitivity is evaluated in the theorem below.

Theorem 5.1. Consider the error equation (3.6). Then the sensitivity of the performance index (5.1) with respect to variations of the adaptation gain γ at $\gamma=0$ is given by

$$J_\gamma=-4\ \text{avg}\{e_0H_0(p)[\bar{\phi}_0^TH_1(p)[\bar{\phi}_0e_o(t)]]\}. \qquad (5.4)$$

□□□

Remark 5.1. The condition for performance improvement (5.3) requires the operator

$$e_0(t)\rightarrow H_0(p)[\bar{\phi}_0^TH_1(p)[\bar{\phi}_0e_0(t)]] \qquad (5.5)$$

to be "sign-preserving in average", i.e., a passivity condition. Satisfaction of this condition for all possible signals $e_0(t)$, $\bar{\phi}_0(t)$ requires $H_0(p)H_1(p)$ to be strictly positive real. Therefore, is unattainable in practice. Notice, however, that failure to establish the passivity of the operator does not imply that performance is degraded with the adaptive tuner.

To get some insight into the nature of condition (5.3) let us consider the simplest case of one adjustable parameter and assume that

$$\bar{\phi}_0(t)=\alpha_\phi\cos(\omega_\phi t) \qquad (5.6a)$$

$$e_0(t)=\alpha_e\cos(\omega_c t),\ \omega_e\neq\omega_\phi. \qquad (5.6b)$$

Some simple but lengthy calculations show that in this case

$$J_\gamma=-\alpha_e^2\alpha_\phi^2\alpha_0[\alpha_+\cos(\Phi_++\Phi_0)+\alpha_-\cos(\Phi_-+\Phi_0)] \qquad (5.7)$$

where

$$\alpha_+e^{j\phi_0}=H(j\omega_e) \qquad (5.8)$$

$$\alpha_+e^{j\phi_+}=H_1[(j(\omega_e+\omega_\phi)]; \quad \alpha_-e^{j\phi_-}=H_1[(j(\omega_e-\omega_\phi)]$$

Notice that ϕ_+, ϕ_- in (5.7) are due to the phase shift contribution of $H_1(p)$ when acting on the signal $\bar{\phi}_0(t)e_0(t)$. If we assume that σ_0 is sufficiently large, in the sense that the phase shift contribution of $H_1(p)$ is negligible in the frequency range of interest, we can approximate (5.4) by

$$-J_\gamma\approx 4\text{avg}\{e_0(t)H_0(p)[\|\bar{\phi}_0(t)\|_E^2e_0(t)]\}. \qquad (5.9)$$

The requirement of large σ_0 is consistent with the conditions for preservation of stability of the previous section. With this approximation a simple condition for performance improvement can be easily derived from (5.9) as follows:

Assume $\vec{\phi}(t)$, $e_0(t)$ almost periodic with generalized Fourier series (4.9) and

$$e_0(t) \sim \sum_{\omega\in\Omega_e} \alpha(\omega) e^{j\omega t} \quad (5.10)$$

respectively. Assume further that

$$\Omega_e \cap \Omega = \phi. \quad (5.11)$$

Then, J_γ as given in (5.9), is negative if

$$\sum_{\omega\in\Omega_e} \|\alpha(\omega)\|_E^2 R_e\{H_0(j\omega)\} > 0. \quad (5.12)$$

Thus, performance will be **improved** with the adaptive tuner if the amount of "$R_e\{H_0(j\omega)\}$-weighted energy" of $e_0(t)$ in the range of frequencies where $R_e\{H_0(j\omega)\}>0$ is larger than in the range where $R_e\{H_0(j\omega)\}<0$. It is important to remark that this condition is imposed on the tracking error of the plant in closed loop with the **fixed** controller. Therefore, it may be verified before plugging in the adaptation, provided some information on $H_0(j\omega)$ and $e_0(j\omega)$ is available *a priori*.

VI. CONCLUDING REMARKS

The problem of on line tuning of the controller parameters to improve performance for coarsely known plants has been addressed in this paper. The controller structure and number of adjustable parameters is determined by the designer. We require the knowledge of a stabilizing parametrization for the controller but otherwise impose no additional restrictions on the plant except linearity.

A fundamental modification introduced in the paper is the utilization of a new (ρ-modified) parameter estimator. For this ρ-modified update law we have studied conditions under which the inclusion of the on line tuning procedure does not destabilize the otherwise stable closed loop system. Specifically we derived conditions to preserve L_∞-stability, slow adaptation stability /instability and exponential stability. Both the L_∞ and exponential stability result are global on the initial conditions and external references and they don't rely on persistency of excitation arguments.

Besides the exponential stability property mentioned above, performance of the adaptive system is evaluated in two different ways: First, showing that the sup value of the tracking error is bounded from above by a constant times the sup value of the error obtained with the fixed controller. Second, giving conditions under which the RMS performance index decreases when adaptation is turned on.

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APPENDIX

Lemma A.1. (See also [1], [12]). Consider the three input m output system (2.3) with state realization

$$\dot{x}_F(t) = A_F x_F(t) + B_F \begin{vmatrix} u(t) \\ y(t) \\ r(t) \end{vmatrix}, \quad x_F(0) \quad (A.1a)$$

$$\phi(t) = C_F x_F(t) \quad (A.1b)$$

where A_p is Hurwitz with different eigenvalues and (A_F, B_F, C_F) is minimal. Let

$$\dot{\rho}(t) = -\mu\rho(t) + f \quad (A.2a)$$

$$f = \delta[\,|u(t)| + |y(t)| + |r(t)|\,] + 1 \quad (A.2b)$$

where we choose

$$\delta \geq \|C_F\| \; \|B_f\| \quad (A.3a)$$

$$\rho(0) \geq \|C_F\| \; \|x_F(0)\| \quad (A.3b)$$

$$\mu \leq \min_i |Re(\lambda_i)| \quad (A.3c)$$

with λ_i the eigenvalues of A_F. Then

$$\frac{\|\phi(t)\|}{\rho(t)} \leq 1, \forall t \in R_+ \quad (A.4)$$