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INDIRECT ADAPTIVE NONLINEAR CONTROL

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ABSTRACT *"How to make a non-linear control law adaptive ?" Supposing the uncertainties interfere linearly, we answer to this question, for a certain class of systems. This is an illustration/improvement of [2], in which the complete proofs may be found.*

1. INTRODUCTION

We consider a family of non-linear systems with state x in $\mathbf{R}^n$ and input u in $\mathbf{R}^l$, indexed by the parameter vector p :

$$J(p,x)\,\dot{x} = f(p,x) + u\; g(p,x) \tag{1}$$

The maps involved here are smooth with respect to x and linear with respect to p. More precisely (with looseness in the notations):

$$\begin{aligned} J(p,x) &= J_s(x) + J_p(x).p \\ f(p,x) &= f_s(x) + f_p(x).p \quad , \quad f(p,0)=0 \\ g(p,x) &= g_s(x) + g_p(x).p \end{aligned} \tag{2}$$

The parameter vector p lies in Π, a *smooth bounded closed convex* subset of $\mathbf{R}^q$ matching the following

BASIC ASSUMPTIONS :

B1 : $J(p,x)$ is uniformly invertible in $\Pi\times\mathbf{R}^n$:

$$\| J(p,x)^{-1} \| \leq k \quad \text{for any } (p,x) \text{ in } \Pi\times\mathbf{R}^n \tag{3}$$

B2 : For each system (1), i.e. each p in Π, there exists a smooth control $u = v(p,x)$ making the origin a global attractor for :

$$\dot{x} = F(p,x) \triangleq J(p,x)^{-1}\left[\, f(p,x) + v(p,x)\, g(p,x) \,\right] \quad . \tag{4}$$

In fact, there exists a smooth comparison function $V(p,x)$ defined in $\Pi\times\mathbf{R}^n$, such that:

1- $V(p,x)$ is real non-negative and is zero if and only if $x=0$.

2- For any positive M, the following set is bounded: $\{(p,x)\;/\;V(p,x)<M\;,\; p\in\Pi \text{ and } |\,p\,|\leq M\}$.

3- There exists a strictly positive real number, a, such that,

$$\frac{\partial V}{\partial x}.F \leq -a\;V \quad . \tag{5}$$

Functions J, f, g, v and set Π being known, the problem consists in designing a controller which guarantees state regulation and solution boundedness when placed in feedback with one of the systems (1) corresponding to the value p^* of the parameter vector, assumed to be unknown.

This has been done in some particular cases, using various methods:

- In [4], an adaptive controller is proposed, which works in the case when J is the identity matrix, all the systems of the family are linearizable via feedback and diffeomorphism and all the part of the fields actually depending on p are contained in the subspace spanned by the input fields. The control law is $u=v(\hat{p},x)$ [where $v(p,.)$ is feedback linearization] and $\dot{\hat{p}}$ is designed to make a global Lyapunov function decrease.
- For robot arms with as many motors as axis, [1] updates $\hat{p}$ by means of linear estimation and adds some terms to the state-feedback linearization (or computed torque) control law.
- For the same robot arms, [3] achieves the objective with a P.D. control law, updating $\hat{p}$ to make a global positive function decrease.
- In [2], we deal with a general family (1) by means of linear estimation. Global convergence is obtained under constrictive assumptions on the growth at infinity of the open-loop fields.

Here, we propose an estimation-based adaptive controller with corrective terms for more general (1)-families than robot-arms. Assuming some structural properties (T2 below), we obtain global convergence under far less restrictive growth assumptions than in [2]. This paper also helps one to understand which properties of the robot arms allows [1] to work.

TECHNICAL ASSUMPTIONS:

T1 (growth at infinity) : $k(p)$ being a positive continuous function, k and d, α, β, γ, δ positive constants, we assume, for all (p,x) in $\Pi\times\mathbf{R}^n$,

$$\alpha < 1 \tag{6}$$

$$\left|\frac{\partial V}{\partial x}(p,x)\right| \leq k(p)\,\mathrm{Sup}\,\{\,1\,,\,V(p,x)^{\alpha}\,\} \tag{7}$$

$$\left|\frac{\partial V}{\partial p}(p,x)\right| \leq k(p)\,\mathrm{Sup}\,\{\,1\,,\,V(p,x)^{\beta}\,\} \tag{8}$$

$$\left.\begin{aligned} &|\,f_s(x)\,|\;,\;|\,v(p,x)\,g_s(x)\,| \\ &|\,f_p(x)\,|\;,\;|\,v(p,x)\,g_p(x)\,| \end{aligned}\right\} \leq k(p)\,\mathrm{Sup}\,\{1\,,\,V(p,x)^{\gamma}\} \tag{9}$$

$$\left|J(p,x)^{-1}J_p(x)\right| \leq k(p) \tag{10}$$

$$\begin{aligned} &|\,F(p,x) - F(p,y)\,| \\ &\qquad \leq k(p)\,\mathrm{Sup}\,\{\,1\,,\,V(p,x)^{\delta}\,\}\,\Big(1+|\,x-y\,|^{d}\Big)\,|\,x-y\,| \end{aligned} \tag{11}$$

Comment : T1 is true if, for exemple, V is convex and upper and lower-bounded by some polynomials, f and J are upperbounded by some polynomials, and F is a polynomial at infinity.

T2 : There exists G, a linear subspace of $\mathbf{R}^n$, such that, for all (p,x) in $\Pi\times\mathbf{R}^n$,

$$\begin{aligned} &g_p \equiv 0 \quad ; \quad G \subset \mathit{range}\; g_s \\ &J(p,x)\,.\,G = G \\ &\mathit{range}\; J_p \subset G \quad ; \quad \mathit{range}\; f_p \subset G \\ &G \subset \mathit{Ker}\;\frac{\partial J_s}{\partial x}(x) \quad ; \quad G \subset \mathit{Ker}\;\frac{\partial J_p}{\partial x}(x) \end{aligned} \tag{12}$$

2. OUR ADAPTIVE CONTROLLER

2.1. The estimation algorithm

Since $\dot{x}$ is not usually measured, we define the quantities z and Z to perform estimation (this is usually done in the linear case). From assumption T2, $\phi \triangleq \frac{\partial J_s}{\partial x}\dot{x}$ and $\Phi \triangleq \frac{\partial J_p}{\partial x}\dot{x}$ do not depend on p^*; in fact, they are functions of x, $\hat{p}$ and u only. We compute z and Z as follows:

$$\left.\begin{aligned}
\dot{\omega}+\frac{1}{\epsilon}\omega &= \frac{1}{\epsilon}J^{-1}(\hat{p},x)[\phi x-(J_p\dot{\hat{p}}+\phi+\hat{p}\,\Phi)J^{-1}(\hat{p},x)J_s x+f_s+ug_s] \\
\dot{\Omega}+\frac{1}{\epsilon}\Omega &= \frac{1}{\epsilon}J^{-1}(\hat{p},x)[\Phi x-(J_p\dot{\hat{p}}+\phi+\hat{p}\,\Phi)J^{-1}(\hat{p},x)J_p x+f_p+ug_p] \\
z &= \frac{1}{\epsilon}J^{-1}(\hat{p},x)J_s x+\omega \\
Z &= \frac{1}{\epsilon}J^{-1}(\hat{p},x)J_p x+\Omega
\end{aligned}\right\} \quad (13)$$

where ϵ is a positive real number. (1) then yields:

$$z(t) = Z(t)p^* + [\,z(0) - Z(0)p^*\,]\,e^{-t/\epsilon} \quad . \qquad (14)$$

We base linear estimation of p^* on (14), and we choose the following generalization of least square algorithm, $\hat{p}$ being the estimate of p^*:

$$e = z - Z\hat{p} \qquad (15)$$

$$\left.\begin{aligned}
\dot{\hat{p}} &= |\,e\,|^{m-1}\,\pi_{\hat{p}}\left(PZ^T e\right) \quad ; \hat{p}(0)\in\Pi \\
\dot{P} &= -\,|\,e\,|^{m-1}PZ^T ZP \quad ; P(0)=I
\end{aligned}\right\} \quad (16)$$

where:

- $\pi_{\hat{p}}$ is the identity map if $\hat{p}$ lies inside Π and the "P^{-1}-orthogonal projection" onto the boundary of Π if $\hat{p}$ is on this boundary.
- m, larger or equal to 1, is a degree of freedom of the algorithm.

As more or less well known in the case $m=1$ (least squares), this estimation algorithm has the following properties:

Lemma : *Regardless of the control law actually used, i.e. regardless of the function $u(t)$, any solution of the closed-loop system defined on the time interval $[\,0\,,t_f\,)$ (maybe $t_f=\infty$) is such that $\hat{p}(t)$ is bounded and remains in Π, $P(t)$ is a decreasing positive definite matrix, and*

$$\dot{\hat{p}}(t)\in L^1(0,t_f) \qquad e(t)\in L^{m+1}(0,t_f) \quad . \qquad (17)$$

2.2. The control law:

Assumption T2 enables one to find w such that, at any time,

$$\epsilon Z\dot{\hat{p}} + J(\hat{p},x)^{-1}wg(\hat{p},x)\ (t) = Z(0)e^{-\frac{t}{\epsilon}}\dot{\hat{p}}(t) \quad . \qquad (18)$$

u is then given by:

$$u = v(\hat{p},x) + w \qquad (19)$$

Our complete adaptive controller is (19), (13), (15), (16). It is in feedback with:

$$\dot{x} = J(p^*,x)^{-1}\left[f(p^*,x) + u\ g(p^*,x)\right] \qquad (20)$$

3. CONVERGENCE

The state of the closed-loop system is $(x,\hat{p},P,\omega,\Omega)$. The complete behaviour is specified in:

Theorem: *Suppose assumptions* B1, B2, T1, T2 *are true and, in* (16), m *is chosen such that:*

$$m \ \geq\ \text{Sup}\left\{1\,,d\,,\gamma-1\,,\frac{\gamma}{1-\alpha}-1\right\} \qquad (21)$$

Under these conditions, the closed loop system (13), (15), (16), (18), (19), (20) *has the following properties:*

- if $\alpha+\delta\leq 1$ and $\beta\leq 1$, all its solutions are defined and bounded on $[0,+\infty)$, *and x, as well as e and $\hat{p}$, tend to zero.*

- if $\alpha+\delta>1$ or $\beta>1$, the above conclusion holds true only for some initial conditions. If we choose $\Omega(0)$ and $\omega(0)$ in (13) such that $z(0)=\frac{1}{\epsilon}x(0)$ and $Z(0)=0$, The condition is:

$$\frac{m+1}{2}\,|\,p^*-\hat{p}(0)\,|^2 + \epsilon\frac{|\,x(0)\,|^{m+1}}{m+1} \leq C$$

The smaller Sup $\{\alpha+\delta-1,\beta-1\}$, the larger C.

Sketch of proof : If we define y as follows:

$$y(t) = x(t) - \epsilon\, e(t) \quad , \qquad (22)$$

then (4), (13), (15) and (19) yield, omitting t as an argument,

$$\begin{aligned}\dot{y}(t) &= F(\hat{p},y) + \left[F(\hat{p},x)-F(\hat{p},y)\right] \\ &\quad + e + \epsilon Z\dot{\hat{p}} + J(\hat{p},x)^{-1}\,w\ g(\hat{p},x)\end{aligned} \quad , \qquad (23)$$

w is designed to cancel the $\epsilon Z\dot{\hat{p}}$ term (see (18)). Relying on (17), (18), (23) and assumption T1, one may prove that y is bounded. Then, it is not difficult to prove the theorem. A complete very similar proof may be found in [2].

Comments on the assumptions : This is an improvement of [2] since no further constraint is imposed to γ, i.e. to the open-loop fields, to obtain global convergence. Global convergence is for instance obtained if B1 and B2 are true (which is our basic working assumption), T2 is true (this the case of mechanical systems as robots), and, for exemple, f and J are upperbounded by some polynomials and F is linear. This constitutes a result rather different from [4], since our assumption T2 is stronger than these in [4], but we allow implicit linear parametrization, whereas in [4], J is the identity matrix (or does not depend on p).

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