MANAGING CONSTRAINTS FOR INFEASIBLE ROCKET LANDING PROBLEMS

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ABSTRACT

For reusable launchers, in-flight landing trajectory planning is a challenging task. The naturally disturbed atmospheric environment may render the landing objectives – such as the landing site location or the incidence safety bounds – mutually incompatible. This article discusses a methodology to recover feasibility in over-constrained optimization problems used for Powered Descent Guidance. Nominal and emergency guidance problems are first discussed from a high-level viewpoint, and then developed to handle a Quadratic Programming based guidance method, which is illustrated on numerical examples for a planar rocket model.

1. INTRODUCTION

Powered landing for reusable launchers occurs in a naturally disturbed and uncertain environment. For instance, unexpected wind gusts can push a rocket far from the nominal landing site. The online Powered Descent Guidance (PDG) problem, aiming at providing a landing trajectory, must answer the following question at many instants during the flight: provided the current rocket position, orientation, and velocities, is landing still feasible at the landing site, and if not, what is a satisfactory solution?

Though the PDG problem is formulated in several different ways in the literature [1]–[7], a general common feature of the formulation is that they start by solving a feasibility problem for a constrained dynamical system. We are interested in the landing scenarios when this problem is not feasible anymore – as shown in Figure 1. Our work focuses on designing a method to keep providing a landing trajectory when the ideal target is not reachable anymore. Such a trajectory is called an *emergency trajectory*, and is computed by relaxing a constraint subset, using *negotiable parameters*.

This article is structured as follows. First, a high-level description of the nominal and emergency guidance scenario is introduced in Section 2. More precisely, a systematic method – see Algorithm 1 below – chooses the constraints that can or cannot be loosened. In Section 3, we show that if the initial PDG problem is formulated as a Quadratic Program (QP) – using previous work of the authors [8] – then the negotiability problem can be formulated as a Linear Program (LP). The latter makes real-time resolution possible by relying on off-the-shelf solvers. Numerical simulations are provided in Section 4, to demonstrate the above-mentioned concepts for various choices of negotiable parameters.

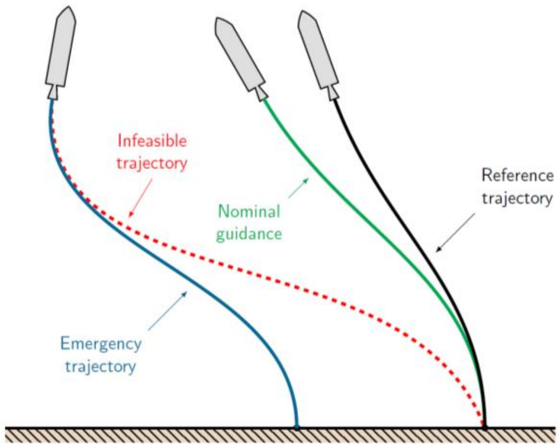

Figure 1: The need for *emergency* guidance. Here, it is impossible to design a landing trajectory satisfying all the objectives for the left-most rocket initial conditions.

2. THE EMERGENCY PROBLEM

First, the variables describing the PDG problem are defined, and a generic description of nominal guidance method is introduced. Then, a high-level description of a procedure providing emergency guidance is presented.

2.1. Powered Descent Guidance variables

As will be detailed below, the PDG problem is described by its decision variable z, its input ξ, and several objective parameters p.

Consider a rocket described by its state x, conveying its positions, its speeds, and its mass. It is assumed that it is controlled by a variable u, conveying its engine flow q, and its incidence α.

The inputs of the PDG problem are the initial conditions of the rocket – denoted x^0 – and parameters – denoted η – that influence the Equations of Motion (EoM), such as the wind magnitude (w) or the Engine Specific Impulse (Isp). The dynamic equation writes

$$\dot{x} = f(x, u, \eta).$$

To alleviate the writing, these inputs are conveyed by a variable $\xi = (x^0, \eta)$.

We consider that u, which is an infinite dimensional variable in general, is approximated by a finite dimensional variable μ. Basically, μ conveys the values of u at some prescribed collocation points [8], [9].

Another important variable for the PDG problem is the time-of-flight t_f which is implicitly defined by the landing condition constraint "Altitude$(t_f) = 0$." Therefore, the finite-dimensional decision variable of our generic PDG problem is

$$z = \begin{pmatrix} \mu \\ t_f \end{pmatrix}.$$

Finally, the PDG problem has several objective-related parameters, conveyed by a variable p. For instance, the position of the landing site or the incidence bound (respectively denoted z^f and $\alpha_{\max}$) are problem-specific choices that can be present in this parameter p. Two examples will be presented in Section 4. The value of these parameters is p_{nom} for a nominal landing scenario.

2.2. The nominal PDG problem

Let us denote by $\mathcal{F}(\xi, p)$ the set of decision variables z that satisfy the constraints above. Basically, $\mathcal{F}$ conveys the dynamic equation of f, the various bounds on the engine and the incidence, the landing site location constraint, and other bounds such as normal load limits. Also, denote by $J(z)$ a cost on z that defines how *desirable* is the trajectory defined by z. Then, the problem of finding the best landing trajectory boils down to solving

$$\text{PDG}(\xi) \quad := \quad \min_{z \in \mathcal{F}(\xi, p_{\text{nom}})} J(z)\,.$$

Most of the current research efforts in PDG focus on finding the best way to express this problem and using the fastest and most reliable computational methods to solve it [1], [3], [7].

$\text{PDG}(\xi)$ is what we call the *nominal guidance* problem, when $\mathcal{F}(\xi, p_{\text{nom}})$ is a non-empty set. However, it is possible that for some values of ξ, the latter becomes infeasible, i.e., $\mathcal{F}(\xi, p_{\text{nom}}) = \emptyset$. Altering the value of p to recover feasibility is what we call *emergency* PDG. Such a procedure is presented below.

2.3. A generic methodology for emergency guidance

Consider an infeasible case, and let us formulate a question worth answering, e.g., *what should be done if the rocket starts its descent too far from the landing site?* To answer it, we propose a general approach, detailed below, that we apply on a specific landing procedure in the next section.

Emergency landing is required when the given input ξ makes $\mathcal{F}(\xi, p_{\text{nom}})$ empty. This is due to the impossibility to achieve the target values p_{nom}. We propose to first compute the smallest perturbation of p that recovers feasibility, and then to re-optimize it in view of the original $\text{PDG}(\xi)$ problem.

The perturbation in the variable p is denoted Δp such that $p = p_{\text{nom}} + \Delta p$, where Δp is bounded between Δp^- and Δp^+. This perturbation Δp represent our *negotiable parameters*. Thus, we can introduce $\text{Negotiate}(\xi)$, also called the negotiation problem, which is

$$\begin{aligned} \min_{z, \Delta p} \quad & \gamma(\Delta p) \\ \text{s.t.:} \quad & z \in \mathcal{F}(\eta, p_{\text{nom}} + \Delta p), \\ & \Delta p^- \le \Delta p \le \Delta p^+. \end{aligned}$$

for some arbitrary non-negative penalty function γ satisfying

$$\gamma(\Delta p) = 0 \quad \Leftrightarrow \quad \Delta p = 0.$$

Typically, γ is a norm such as the 1-norm or the infinity norm. We are interested in the optimal value of $\text{Negotiate}(\xi)$, denoted $\mathcal{P}^*$, which allows us to define the problem $\text{Refine}(\xi, \mathcal{P}^*)$ such that

$$\begin{aligned} \min_{z, \Delta p} \quad & J(z) \\ \text{s.t.:} \quad & z \in \mathcal{F}(\xi, p_{\text{nom}} + \Delta p), \\ & \Delta p^- \le \Delta p \le \Delta p^+, \\ & \gamma(\Delta p) = \mathcal{P}^*. \end{aligned}$$

The fact that Δp is free in the constraints of $\text{Refine}(\xi, \mathcal{P}^*)$ means that we seek the optimal value of z such that the objective parameters are as close as possible – in the sense imposed by γ – from their nominal value p_{nom}. The high-level procedure for Emergency Powered Descent Guidance (E-PDG) is described by Algorithm 1.

Input: ξ
Step 1: Solve $\text{Negotiate}(\xi)$, which gives an optimal penalty $\mathcal{P}^*$on the negotiable parameters.
Step 2: Solve $\text{Refine}(\xi, \mathcal{P}^*)$, which gives an optimal landing strategy z^*.
Output. z^*

Algorithm 1: Emergency Powered Descent Guidance (E-PDG)

Note that this algorithm *does not* expect $\text{Negotiate}(\xi)$ to have a unique solution (otherwise Step 2 is useless), since only the value of $\mathcal{P}^*$ matters.

Moreover, E-PDG is compatible with the nominal guidance. In other words, for all the values of ξ for which $\text{PDG}(\xi)$ is feasible, E-PDG and $\text{PDG}(\xi)$ will return the same value. Indeed, if $\mathcal{F}(\xi, p_{\text{nom}})$ is feasible for some input ξ, then $\text{Negotiate}(\xi)$ will return $\mathcal{P}^* = 0$, which implies $\Delta p = 0$. In this case, $\text{Refine}(\xi, 0)$ is directly equivalent to $\text{PDG}(\xi)$. This is one salient advantage of E-PDG: it can be *plugged* in place of any arbitrary nominal guidance method.

3. QP/LP EMERGENCY GUIDANCE

In practice, we applied the E-PDG algorithm to a Quadratic Programming based guidance method developed by the authors [8].

3.1. PDG using Quadratic Programming

Extending the notations from [8], the main problem $\text{PDG}(\xi)$ is described using a quadratic cost and linear constraints, which we will denote $\text{QP}(\xi)$

$$\begin{aligned} \min_{z} \quad & \frac{1}{2} z^\top W z \\ \text{s.t.:} \quad & Gz \leq h + H_p\, p_{\text{nom}} \\ & Az = b + B_p\, p_{\text{nom}} \end{aligned}$$

where G, h, H, A, b, B are matrices that depend on a mission-specific reference trajectory. Here, h and b are affine functions of the input ξ. Here, W simply denotes a weighting matrix, assumed positive-definite.

3.2. Application of E-PDG

The penalty γ is here the 1-norm. The negotiation problem is denoted $\text{Negotiate}_{\text{LP}}(\xi)$ and consist in

$$\begin{aligned} \min_{z, \Delta p} \quad & \|\Delta p\|_1 \\ \text{s.t.:} \quad & Gz \leq h + H_p\, (p_{\text{nom}} + \Delta p) \\ & Az = b + B_p\, (p_{\text{nom}} + \Delta p) \\ & \Delta p^- \leq \Delta p \leq \Delta p^+ \end{aligned}$$

and returns its optimal value $\mathcal{P}^*$. The refine problem is denoted $\text{Refine}_{\text{QP}}(\xi, \mathcal{P}^*)$ and is

$$\begin{aligned} \min_{z, \Delta p} \quad & \frac{1}{2} z^\top W z \\ \text{s.t.:} \quad & Gz \leq h + H_p\, (p_{\text{nom}} + \Delta p) \\ & Az = b + B_p\, (p_{\text{nom}} + \Delta p) \\ & \Delta p^- \leq \Delta p \leq \Delta p^+ \\ & \|\Delta p\|_1 = \mathcal{P}^* \end{aligned}$$

Note, as suggested by its name, that $\text{Negotiate}_{\text{LP}}(\xi)$ can be solved using Linear Programming, by decomposing Δp into its positive and negative parts. See Example 1.13 in Betts' book [10] for further details. Likewise, $\text{Refine}_{\text{QP}}(\xi, \mathcal{P}^*)$ is solved using Quadratic Programming [11].

It is noteworthy that the problem $\text{Negotiate}_{\text{LP}}(\xi)$, aiming at recovering feasibility, builds upon mathematical programming techniques known as *right-hand side alteration methods* [12]. Here, as conveyed by the matrices H_p and B_p and due to the limited number of levers, only a sub-space of the constraint right-hand side can be altered.

3.3. A geometric interpretation

The connections between problems $\text{QP}(\xi)$, $\text{Negotiate}_{\text{LP}}(\xi)$ and $\text{Refine}_{\text{QP}}(\xi, \mathcal{P}^*)$ can be easily understood from a geometric point of view. On a toy-problem, let us see with a simplifying perspective how these work by projecting the constraint on a planar representation.

The constraints of $\text{QP}(\xi)$ define a polytope: the inequalities define half-spaces, and the equalities define hyperplanes. Thus, the feasible set for the variable z in $QP(0)$ is the set represented in blue in Figure 2.

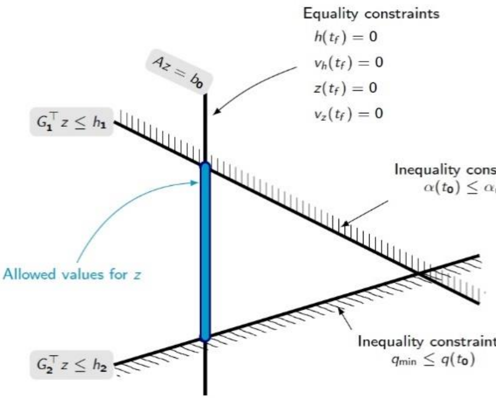

Figure 2: The constraints of $\text{QP}(0)$.

When the inputs are non-zero, for example when $\Delta x^0 \neq 0$, the right-hand side vectors h and b of the constraints of $\text{QP}(\xi)$ are modified. Geometrically, it corresponds to a pure translation of the lines representing the constraints, as shown in Figure 3.

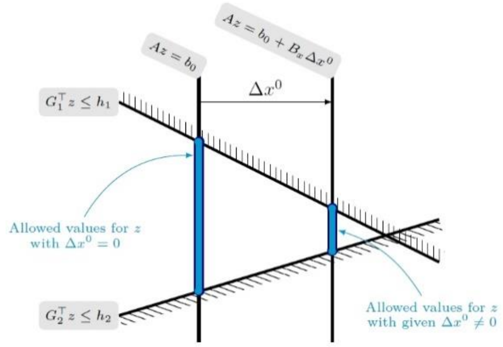

Figure 3: The constraints of QP(ξ),when $\Delta x^0 \neq 0$.

However, if Δx^0 is too large (for example when the rocket starts its descent too far from the landing site, as pictured in Figure 1), then satisfying all the constraints simultaneously becomes impossible, as Figure 4 suggests.

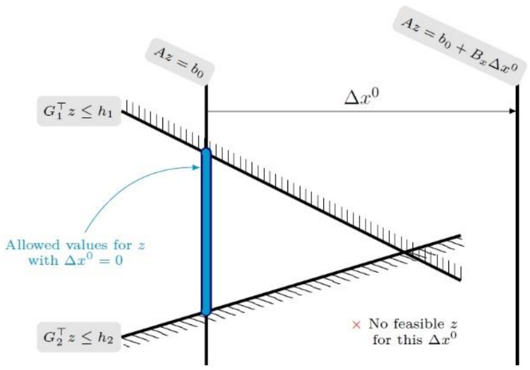

Figure 4: The constraints of QP(ξ),when Δx^0 makes the constraints infeasible.

Hence the need for the E-PDG algorithm. Let us assume that the final horizontal position Δz^f can be negotiated, i.e., we take $p = \Delta z^f$. Modifying its value will have a direct impact on the right-hand side of the equality constraints from QP(ξ). By solving $\text{Negotiate}_{\text{LP}}(\xi)$, we find the smallest value of Δz^f that recovers feasibility, as illustrated in Figure 5.

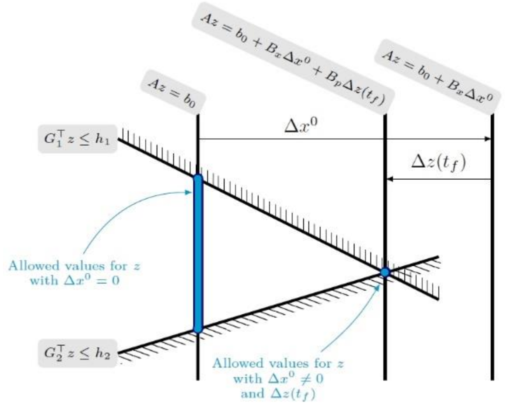

Figure 5: The constraints of QP(ξ), when $\Delta p = (\Delta z^f)$ helps recover feasibility.

4. NUMERICAL SIMULATIONS

Let us apply the above-mentioned guidance algorithm, to two scenarios conveying different negotiation parameters.

Both scenarios presented below consider inputs defined with respect to a reference trajectory, shown in **plain black** in Figures 6 and 7. Several input values ξ that differ in their initial horizontal position Δz^0 are presented. Note that to ease the comparison between them, Figures 6 and 7 are plotted for the same input list.

4.1. FINAL HORIZONTAL POSITION

Let us assume that the landing site is very large (for instance if the rocket landing site is in a flat desert). Thus, if necessary, the final horizontal position Δz^f can be negotiated. As shown in Figure 6, if Δx^0 becomes large enough, negotiation is needed, and it yields the farthest trajectories (in dashed blue).

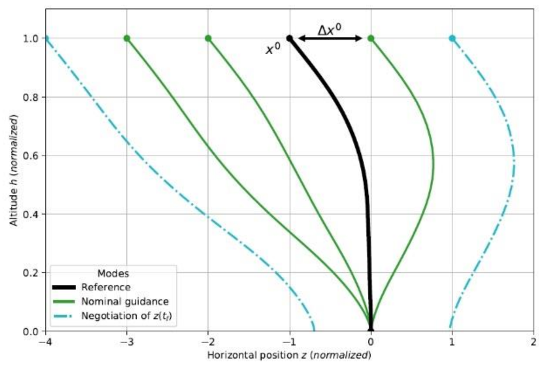

Figure 6: Nominal and emergency trajectories, when landing site location is allowed to be negotiated.

4.2. INCIDENCE BOUND

Contrary to the previous example, let us assume that the final horizontal position is not negotiable, which happens if one whishes to land on a platform at see for instance. Instead, we assume that the safety bound α_{max} imposed on the rocket incidence α, can be negotiated by a parameter $p = \Delta\alpha_{max}$ such that

$$|\alpha| \leq \alpha_{max} + \Delta\alpha_{max}.$$

Considering the same input values as in the previous example, we see that the farthest trajectories from the reference need to negotiate the value of $\Delta\alpha_{max}$ to recover feasibility, leading to a sharped turn for the dashed pink trajectories shown in Figure 7.

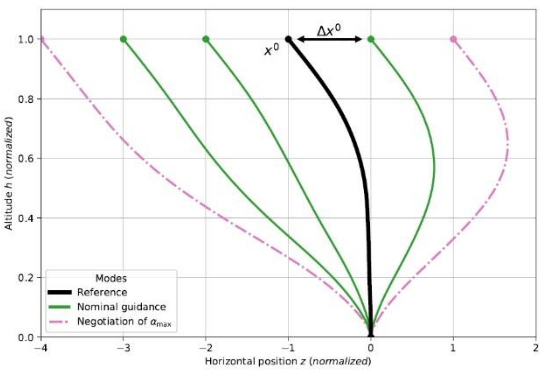

Figure 7: Nominal and emergency trajectories, when the incidence bound is allowed to be negotiated.

5. FUTURE WORK

This paper presented a generic methodology to provide emergency guidance for infeasible Powered Descent Guidance problems and has been illustrated on a 2D rocket model.

Future work will focus on extending the E-PDG algorithm to a richer framework that enforces a hierarchy of importance between the components of p (see for example [13], which has been accepted after presenting our work at the 25th ESA Symposium, but before the final editing of this paper). Moreover, extension to more complex rocket models, with 3D descriptions and non-trivial engine dynamics are under study.

6. REFERENCES

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