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Online Energy Management System (EMS) Including Engine and Catalyst Temperatures for a Parallel HEV

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Abstract: In this paper, a first practical extension of the Equivalent Consumption Minimization Strategy (ECMS) is proposed to include thermal dynamics (engine and catalyst temperatures) in the optimal design of an Energy Management System (EMS) for a parallel Hybrid-Electric light-duty Vehicle (HEV). The task of this novel multi-state ECMS is to achieve a sufficient level of performance with respect to pollutant emissions while keeping fuel consumption within acceptable limits. The extension suggested here is based on correlations between the thermal state and their corresponding adjoint states, observed along extremal calculated from extensive offline solutions of optimal control problems. Simulation results stress that the obtained performance is sufficient to satisfy the environmental norms while keeping fuel consumption sub-optimality relatively marginal.

Keywords: hybrid-electric vehicle, energy management, multi-state ECMS, pollutant emissions minimization, state-to-costate feedback

1. INTRODUCTION

As is largely acknowledged, the Hybrid Electric Vehicle (HEV) technology is a major solution to reduce fuel consumption and pollutant emissions of passenger cars. Having two on-board energy sources provides a valuable degree of flexibility in the power generation. To handle this degree of freedom and to coordinate the components of the powertrain in an efficient manner, Energy Management System (EMS) is commonly employed (Guzzella and Sciarretta, 2013).

The design of an EMS can be conveniently formulated as a dynamic optimization problem (Guzzella and Sciarretta, 2013; Paganelli et al., 2002; Sciarretta et al., 2004). This approach is based on the definition of a cost function to be minimized by a dynamical system representing the vehicle dynamics. A main difficulty in solving such problem, in real time, is the presence of disturbances represented by the vehicle driver's actions (e.g. power requests for traction). These disturbances are highly dynamic (their typical frequency is approx. 1Hz) and difficult to predict, as they clearly depend on many factors such as local traffic, infrastructure status, non-vehicle actors, weather conditions (Paganelli et al., 2002; Chasse et al., 2009). For these two reasons, dynamic optimization approaches based on Model Predictive Control (MPC) techniques, that prove so effective in other applications, usually fail in this context.

The alternative approach, which has emerged in the last years is a prediction-free, costate-to-state feedback approach based partially on Pontryagin Minimum Principle (PMP) (Pontryagin et al., 1962). To be implemented, this approach only needs the current value of the disturbance and not its future profile, at least not so precisely as would be necessary to run a global optimization. In more details, the costate function is estimated on a second-to-second basis as the output of a feedback controller on the state variable (which is assumed to be measured or estimated with good accuracy). Such state-costate dependency is established on a physical basis and experimental observations. Then, the control variable is determined as the minimizer (for each time) of the Hamiltonian of the system (Paganelli et al., 2002; Bryson and Ho, 1969). In the specific literature, the strategy embodying this approach is known as Equivalent Consumption Minimization Strategy (ECMS) (Paganelli et al., 2002; Delprat et al., 2003; Musardo et al., 2005). It applies to an EMS that considers only the battery State of Charge (SoC) as a state variable. Experience suggests that the terminal SoC is a monotonic increasing function of the initial costate value (Paganelli et al., 2002; Chasse et al., 2009; Delprat et al., 2003). When such a dependency is inverted to compute the value of the costate as a function of the SoC, it gives rise to the ECMS. So far, this effective technique has been limited to the aforementioned case where only one state is considered in the optimization.

In numerous studies for EMS of HEV, one frequent hidden assumption is that the HEV system is under thermal equilibrium. However, from an engine modeling viewpoint, engine temperature is an important factor that influences both fuel consumption and pollutant emissions (Kiencke and Nielsen, 2005). The engine is subject to stop-start phases, and its temperature may drop. On the pollutant side, the after-treatment system is only activated beyond a certain threshold temperature, and its efficiency is relatively poor at low catalyst temperatures (Michel et al., 2014b; Lescot et al., 2010; Merz et al., 2012; Serrao et al., 2011; Chasse et al., 2010; Michel et al., 2014a).

In this paper, the objective is to extend the costate-to-state feedback approach to more complex cases with multiple state variables. The considered case is the minimization of a trade-off between pollutant emissions and fuel consumption under no equilibrium thermal state (cold start). The engine and catalyst temperatures has to be considered as state variables (Maamria et al., 2015). The system becomes a multi-state system, and thus the PMP approach requires multiple-costate estimation. The proposed extension is based on the parametrization of the relations between the adjoint state variables and their corresponding states (which are assumed to be measured, or at least well estimated) independently of the driving cycles. These relations can be determined from offline numerical experimentations. The proposed real-time strategies are capable of handling some degrees of uncertainty in the future driving scenarios.

The paper is organized as follows. In Section 2, a mathematical control-oriented model taking into account the influence of engine and catalyst temperatures on fuel consumption and pollutant emissions and the optimization problem under consideration are presented. Then, a PMP solution for the case where the driving cycle is known in advance, and the employed numerical method are described in Section 3. On this basis, the novel multi-state ECMS designed to include the thermal dynamics of the engine and the after-treatment system is presented in Section 4. Finally, the obtained numerical results are presented and discussed in Section 5 and some conclusions and perspectives are drawn in Section 6. An appendix is added where the links with neighboring extremals and the presented extension are sketched.

2. SYSTEM DESCRIPTION AND OPTIMIZATION PROBLEM FORMULATION

2.1 System Description

The parallel hybrid architecture studied in this paper is depicted in Figure 1. The internal combustion engine is fitted with an after-treatment system. The electric motor allows the power assist, including purely electric drive and battery recharge. The transmission ratio between the electric motor and the wheels is constant, while the gearbox is an automated manual transmission. Additionally, a battery is used as an energy storage system for the electric energy.

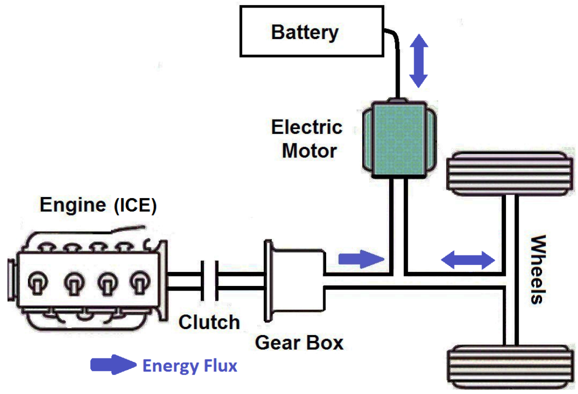

Fig. 1. The chosen parallel HEV architecture

2.2 Optimal Control Problem (OCP) Formulation

The modeling methodology is adopted from (Guzzella and Sciarretta, 2013), resulting in a quasi-static model of the vehicle components. The OCP considered here was studied in (Maamria et al., 2015). We assume that the vehicle follows a prescribed driving cycle.

Cost function The cost function (1) to be minimized is a weighted sum of fuel consumption c and pollutant emissions rate m_{poll} out of the after-treatment system, over a fixed time window corresponding to a driving cycle of a duration T:

$$J(u) = \int_0^T [(1-\alpha)c(u,t,\theta_e) + \alpha m_{poll}(u,t,\theta_e,\theta_c)]dt. \quad (1)$$

In (1), the parameter $0 \le \alpha \le 1$ is a weighting factor serving to adjust the relative importance of fuel consumption and pollutant emissions, u is the engine torque, θ_e is the engine (coolant) temperature, θ_c is the after-treatment system temperature. The time variable t accounts for the dependence of fuel consumption and pollutant emissions on the engine speed, which is a set path defined by the driving cycle to be tracked.

In (1), $c(.)$ is the instantaneous fuel consumption:

$$c(u,t,\theta_e) = c_h(u,t)e(\theta_e),$$

where the function $c_h(.)$ is the fuel consumption rate for a warm engine. It is given by a quasi-steady map as a function of the engine speed and torque (derived from experimental engine tests). The correction factor $e(.)$ of fuel consumption is a decreasing function (not necessarily smooth) with asymptotic value of 1. In our case, $e(.)$ is approximated by

$$e(\theta) = \begin{cases} -a\theta + b, \ \theta_c \le \theta \le \theta_w, \\ 1, \ \theta > \theta_w, \end{cases}$$

where a and b are positive constants, which were identified from experimental data extracted from the engine control maps given by car manufacturers (see (Maamria et al., 2014)). The threshold θ_w is an admissible value of θ_e after which the thermostat is activated (its value is around 80°C).

Similarly, the rate of the pollutant emissions (CO, HC, NO_x) out of the after-treatment system $m_{poll}(.)$ is of the form

$$m_{poll}(u,t,\theta_e,\theta_c) = m_{poll,h}(u,t)e_{poll}(\theta_e)(1-\eta_{poll}(\theta_c)),$$

where $m_{poll,h}$ is the emission rate out of the engine when the engine is warm, given by a quasi-steady map as a function of engine speed and engine torque. The correction factor $e_{poll}(.)$ is a decreasing function of θ_e and is always greater or equal to one. η_{poll} is the after-treatment system conversion efficiency (see Figure 2) which depends on the catalyst temperature θ_c.

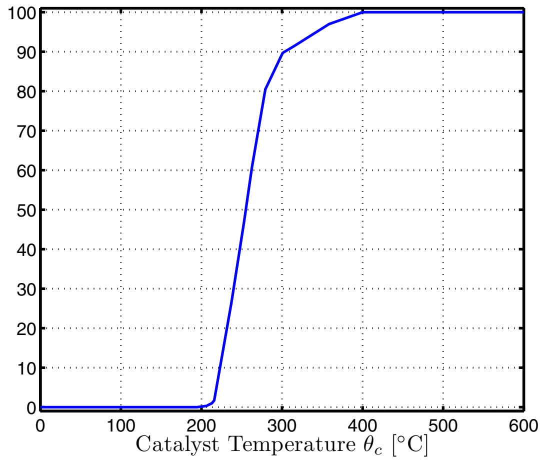

Fig. 2. After-treatment system conversion efficiency for pollutant emission

State variables Three state variables are considered. The first one is the SoC (ξ). Its dynamics is given by

$$\dot{\xi} = -\frac{I_b}{Q_0}, \quad \xi(0) = \xi_0,$$

where I_b is the battery current intensity and Q_0 is the nominal battery capacity (see (Guzzella and Sciarretta, 2013; Padovani et al., 2013) for more details).

One operational constraint for charge-sustaining HEVs requires that the final value of ξ should be equal to its initial value

$$\xi(T) = \xi(0). \tag{2}$$

The dependency of the current I_b on ξ is neglected in the control model as commonly assumed in the literature (Kim et al., 2011). In what follows, the dynamics of ξ considering a given initial condition ξ_0 is written as

$$\frac{d\xi}{dt} = f(u,t), \quad \xi(0) = \xi_0. \tag{3}$$

The second state variable is the engine temperature (θ_e). It satisfies the first order non-linear differential equation (Merz et al., 2012)

$$C_e \frac{d\theta_e}{dt} = P_{th,e}(u,t,\theta_e) - G_e \cdot (\theta_e - \theta_0) - P_{th,aux},$$

where C_e is an equivalent thermal capacity, G_e is an equivalent thermal conductivity, θ_0 is the ambient temperature, $P_{th,e}$ is the sum of friction power dissipated into heat and thermal power transferred from the engine to the coolant (given by a look-up table), and $P_{th,aux}$ is the thermal power drained by the cabin heater (considered constant). In what follows, the dynamics of θ_e considering the initial condition θ_0 as the ambient temperature is written as

$$\frac{d\theta_e}{dt} = g(u,t,\theta_e), \quad \theta_e(0) = \theta_0. \tag{4}$$

The last state variable is the catalyst temperature (θ_c). This variable represents the (spatially averaged) catalyst temperature (Serrao et al., 2011; Eriksson, 2002). The considered model is a zero-dimensional model based on physical equations. Based on an energy balance, the after-treatment system temperature variation can be approximated by

$$C_c(\theta_c) \cdot \frac{d\theta_c}{dt} = P_{th,ec} - P_{th,cr} - G_c \cdot (\theta_c - \theta_0) + P_{ch,c},$$

where C_c is an equivalent thermal capacity of the catalyst depending on θ_c, G_c in an equivalent conductance of the catalyst and θ_0 is the ambient temperature. The term $G_c \cdot (\theta_c - \theta_0)$ represents the heat flux exchanged with ambient air (mainly governed by convection). The term $P_{ch,c}$ is the rate of heat released by the chemical reactions. Details about this model are given in (Maamria et al., 2015; Merz et al., 2012; Michel et al., 2014a; Eriksson, 2002). In order to simplify the notations, the dynamics of θ_c considering a given initial condition $\theta_{c,0}$ is written as

$$\frac{d\theta_c}{dt} = k(u,t,\theta_e,\theta_c), \quad \theta_c(0) = \theta_{c,0}. \tag{5}$$

Constraints The control u is constrained to belong to a set U^{ad} defined by:

$$u_{min}(t) \le u(t) \le u_{max}(t) \tag{6}$$

where the bounds u_{max} and u_{min} are determined by the driving conditions, and physical limitations of the engine and the electric motor.

More generally, the OCP could include some instantaneous constraints on the state variables ξ and θ_e, but this would lead to a more complicated numerical solving. The constraints on θ_e are not important in this study, because the cost function will be independent from θ_e after a certain threshold which is generally an admissible value. On the other hand, and as the vehicle is equipped with a large battery, we assume that the constraints on xi are satisfied. Therefore, the state constraints are omitted.

In summary, the OCP, denoted by (OCP), is defined

$$(OCP) \quad \min_{u \in U^{ad}} J(u), \tag{7}$$

under the dynamics (3, 4, 5) and the boundary constraint (2). The obtained control strategy is noted (S_2) [1].

3. OPTIMAL SOLUTION FROM PMP

Based on the Pontryagin Minimum Principle (PMP) (Pontryagin et al., 1962), the Hamiltonian H is defined by

$$H = L(u,t,\theta_e,\theta_c) + \lambda f(u,t) + \mu g(u,t,\theta_e) + \nu k(u,t,\theta_e,\theta_c)$$

where λ, μ, ν are the adjoint variables associated to ξ, θ_e and θ_c respectively, and L is given by

$$L(u,t,\theta_e,\theta_c) = (1-\alpha) \cdot c(u,t,\theta_e) + \alpha \cdot m_{poll}(u,t,\theta_e,\theta_c).$$

The adjoint states $\lambda(t)$, $\mu(t)$ and $\nu(t)$ are defined by

$$\frac{d\lambda}{dt} = -\frac{\partial H}{\partial \xi} = 0, \; \frac{d\mu}{dt} = -\frac{\partial H}{\partial \theta_e}, \; \frac{d\nu}{dt} = -\frac{\partial H}{\partial \theta_c} \tag{8}$$

with

[1] The subscript $_2$ in S_2 refers to the number of the thermal state variables.

$$\mu(T) = 0,\ \nu(T) = 0, \tag{9}$$

since the final temperatures $\theta_e(T)$ and $\theta_c(T)$ are free and the final time T is fixed. On the other hand, λ is constant and its value will be calculated to satisfy the final SoC constraint (2).

If u^* is an optimal control, then, for every t, $u^*(t)$ minimizes the Hamiltonian in the set defined by (6) along optimal states and corresponding adjoint states trajectories

$$u^* \in \arg\min_{u \in U^{ad}} H(u, t, \theta_e, \theta_c, \lambda, \mu, \nu). \tag{10}$$

Equations (2, 3, 4, 5, 8, 9, 10) constitute a Two-Point Boundary Value Problem (TPBVP). The solution of this TPBVP was done for a pre-defined driving cycle and the results were discussed in (Maamria et al., 2015).

In what follows, for convenience, equivalence factors that are positive and dimensionless, denoted by (s, p, q), are used instead of using (λ, μ, ν) (which are negative). The relationships between them are given by

$$s(t) = -\frac{H_{lhv}\lambda(t)}{Q_0 U_{ocv}},\ p(t) = -\frac{H_{lhv}}{C_e}\mu(t),\ q(t) = -\frac{H_{lhv}}{C_c}\nu(t),$$

where H_{lhv} is the lower heating value of the fuel.

4. TOWARD ONLINE CONTROL STRATEGY: COSTATES ADAPTATION

Although the PMP represents a powerful tool for solving the energy management problems for HEV, it is not suitable for real-time applications because of the dependence of its solution in the future driving conditions. In real situation, the future driving conditions are unknown. For practical applications, the control strategy has to be calculated from the available information in real-time. One of the well-known methods to deal with driving condition uncertainties is the ECMS (Paganelli et al., 2002; Sciarretta et al., 2004; Serrao et al., 2009). This method combines the results of optimal control:

- the control variable minimizes at all times the Hamiltonian of the system in equation (10),
- the adjoint variable is calculated as the output of a feedback controller on the state variables (assumed to be measured or at least estimated) of the form

$$s(t) = s_0 - k_p \cdot (\xi(t) - \xi_{ref}) - k_i \int_0^t (\xi(s) - \xi_{ref}) ds \tag{11}$$

  where s_0 is a first guess possibly inspired by offline calculations, k_p is a tunable positive coefficient, and k_i is a tunable positive coefficient to favor the convergence of $\xi(t)$ toward ξ_{ref}. More advanced feedback laws have been reported in the literature (Serrao et al., 2011; Musardo et al., 2005; Onori and Serrao, 2011; Sivertsson and Eriksson, 2014).

The ECMS in its original formulation was studied and used for a single state EMS design (considering only the SoC dynamics) (Chasse et al., 2009; Kessels et al., 2008; Chasse et al., 2010). In what follows, the relation (11) with $k_i = 0$ is used to update the equivalence factor s:

$$s(t) = s_0 - k_p \cdot (\xi(t) - \xi_{ref})$$

For the problem under consideration, two additional equivalence factors (p and q) have to be considered and they are varying over time. For this, the impact of the engine temperature on q is neglected. The objective is to find relationships (mapping) between (p, q) and the thermal states (θ_e, θ_c) of the form

$$p(t) = p(\theta_e(t), \alpha),\quad q(t) = q(\theta_c(t), \alpha). \tag{12}$$

Ideally, these relations have to be robust against the driving conditions variation. The proposed method is based on post-analysis of the PMP results. For *a given driving cycle*, we proceed as follows:

(1) using the PMP, solve the OCP defined in (7) for various values of α,
(2) for each value of α, approximate the relations between (p, q) and (θ_e, θ_c) by polynomial functions,
(3) define the relationship between the coefficients of polynomial functions calculated in step (2) and α by interpolation.

This tedious work is done *once*, and *offline* for *a given driving cycle*. These relations, once identified, are used and tested for other driving cycles.

Multi-state ECMS The control variable u is calculated by minimizing the Hamiltonian H at each time t

$$u \in \arg\min_{u \in U^{ad}} H(u, t, \theta_e, \theta_c, \lambda(\xi(t)), \mu(\theta_e(t), \alpha), \nu(\theta_c(t), \alpha)),$$

where

$$\lambda(\xi) = -\frac{H_{lhv}}{Q_0 U_{ocv}} \cdot (s_0 - k_p \cdot (\xi(t) - \xi_{ref})),$$

$$\mu(\theta_e, \alpha) = -\frac{C_e}{H_{lhv}} \cdot p(\theta_e(t), \alpha),\ \nu(\theta_c, \alpha) = -\frac{C_c}{H_{lhv}} \cdot q(\theta_c(t), \alpha).$$

The expressions of p and q are obtained off-line as explained above. The obtained on-line strategy is denoted by ($ECMS_2$).

5. NUMERICAL RESULTS

Simulation results are obtained for a parallel hybrid electric vehicle equipped with a gasoline engine and a 3-way catalyst system whose characteristics are listed in Table 1. Look-up tables, derived from experimental tests, for fuel consumption $c_h(.)$, CO emissions $m_{CO,h}(.)$ and electric power $P_e(.)$ are reported in (Maamria et al., 2015). CO emissions only are taken to illustrate the method, as they embody all the relevant features of the system. The choice of CO emissions in the cost function is not restrictive, the same approach can be applied in gasoline engines to NOx and HC.

The parameters of the engine and catalyst temperatures are given in (Merz et al., 2012; Maamria et al., 2014; Maamria, 2015)

Table 1. Vehicle characteristics

Vehicle weight	1932 kg
Engine max. power	92 kW
Electric Motor max. power	42 kW
Battery capacity	5 Ah

Four normalized driving cycles are considered: NEDC (New European Driving Cycle), FTP (Federal Test Procedure), WLTC (Worldwide Harmonized Light vehicles Test Procedures) and FHDS cycles.

5.1 Identification of p

Figure 3 shows the relationship, for a fixed value of α, between p and the engine temperature θ_e on the optimal trajectories for the considered driving cycles. The equivalence factor p is a decreasing (monotonic) function of θ_e with an asymptotic value of zero when $\theta_e > 80$. This decreasing behavior can be seen as the reduction of the fuel energy consumption percentage used to overcome frictions at low values of θ_e and to warm-up the engine.

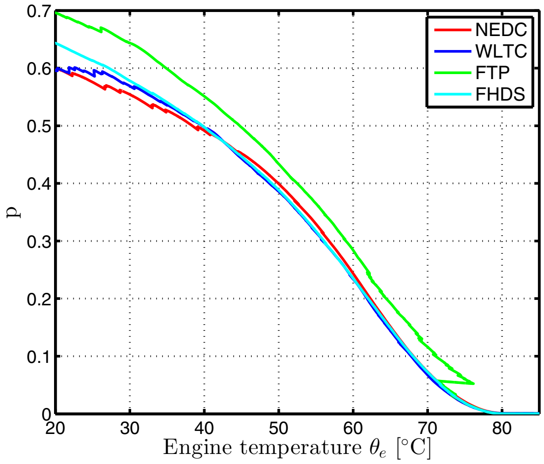

Fig. 3. Equivalence factor p as a function of θ_e

For a fixed value of α, the proposed correlation is

$$p(\theta_e, \alpha) = \begin{cases} d_1(\alpha)\theta_e^2 + d_2(\alpha)\theta_e + d_3(\alpha), \text{ if } \theta_e \leq 80, \\ 0, \text{ if } \theta_e > 80, \end{cases} \tag{13}$$

where the parameters d_i are identified from the optimal solution and for each value of α. The threshold 80°C from which the adjoint state vanishes is supported by the fact that fuel consumption becomes independent of θ_e beyond this value. These parameters, once identified, are tested for other driving cycles to evaluate the robustness of the control strategy.

5.2 Identification of q

Figures 4 and 5 show the relationship between q and θ_c on the optimal trajectories for the considered driving cycles for $\alpha = 0.6$ and $\alpha = 0.8$.

The proposed relationship between q and θ_c for a fixed value of α is of the form

$$q(\theta_c, \alpha) = \begin{cases} h_1(\alpha)\theta_c + h_2(\alpha), \text{ if } \theta_c \leq 200, \\ h_3(\alpha)\theta_c^2 + h_4(\alpha)\theta_c + h_5(\alpha), \text{ if } 200 < \theta_c \leq 400, \\ 0, \text{ if } \theta_c > 400, \end{cases} \tag{14}$$

where the parameters h_i have to be identified from the optimal trajectories. The parameters h_2 and h_5 are determined to provide the continuity of the equivalence factor q at $\theta_c = 200$°C and $\theta_c = 400$°C. The threshold 200°C, beyond which the behavior of the equivalence factor q changes, is the activation temperature of the after-treatment system. The threshold 400°C is the temperature beyond which the efficiency of the after-treatment system reaches its maximum value.

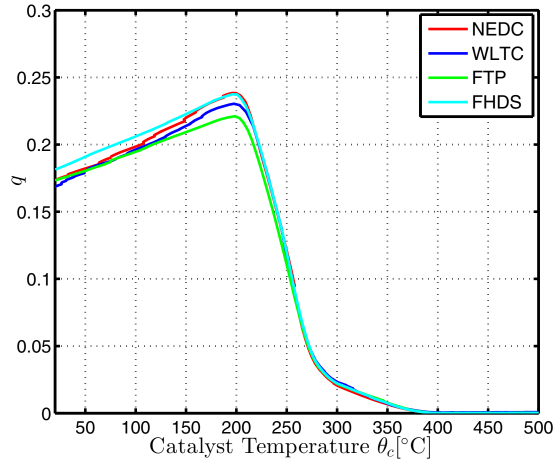

Fig. 4. Equivalence factor q function of θ_c for $\alpha = 0.6$

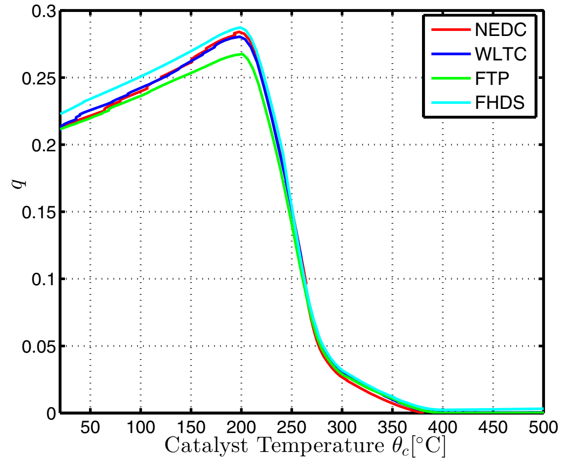

Fig. 5. Equivalence factor q function of θ_c for $\alpha = 0.8$

5.3 Simplified strategies

Two simplified control strategies of (S_2) are presented. The first simplification is to assume that the engine is warm and its temperature θ_e is always greater than $80°C$. This can be formulated

$$e_{CO}(\theta_e) = e(\theta_e) = 1.$$

As the cost is independent from θ_e, only the dynamics of SoC and θ_c have to be considered in the optimization with the final constraint (2). The adjoint variable μ (and thus the equivalence factor p) is null. This simplified strategy is noted (S_1) and its on-line version is denoted by (ECMS$_1$).

An additional possibility to simplify the strategy (S_1) is to assume that the catalyst is never activated and its efficiency η_{poll} is zero:

$$\eta_{poll}(\theta_c) = 0.$$

This simplification is equivalent to minimizing the pollutant emissions out of the engine. As fuel consumption and the pollutant emissions are independent from θ_c, the number of states can be reduced to 1. Only the dynamics of SoC has to be considered with the final constraint (2). The adjoint variables μ and ν (and thus the equivalence factors p and q) are null. We note this simplified strategy (S) and its on-line version by (ECMS).

The control strategies are summarized in Table 2 where the assumptions used to determine optimal trajectories are recalled. *Note that these simplified strategies are only used to compute the control trajectories. For a fair comparison between the various obtained strategies, we use the full model given by equations (1, 3, 4, 5).*

Table 2. Control Strategies

Strategy	Equivalence factors	Assumptions	off/on-line
S_2	$s(t)$, $p(t)$, $q(t)$	/	Offline
S_1	$s(t)$, $q(t)$	$\theta_e \geq 80$	Offline
S	$s(t)$	$\theta_e \geq 80$, $\eta_{poll} = 0$	Offline
$ECMS_2$	$s(\xi)$, $p(\theta_e)$, $q(\theta_c)$	/	On-line
$ECMS_1$	$s(\xi)$, $q(\theta_c)$	$\theta_e \geq 80$	On-line
ECMS	$s(\xi)$	$\theta_e \geq 80$, $\eta_{poll} = 0$	On-line

5.4 Simulation Results

The proposed mapping of $(p,\ q)$ as functions of $(\theta_e,\ \theta_c)$ are determined for the NEDC for various values of α and they will be tested below for the other three driving cycles: WLTC, FTP and FHDS. For the equivalence factor s, a single calibration, for each value of α, of the parameters s_0 and k_p is used. The relative errors between the two strategies (S_2) and ($ECMS_2$) in fuel consumption, CO emissions and the satisfaction of the final SoC constraint (expressed by $\xi(T) - \xi(0)$) are summarized in Figures 6, 7 and 8 respectively.

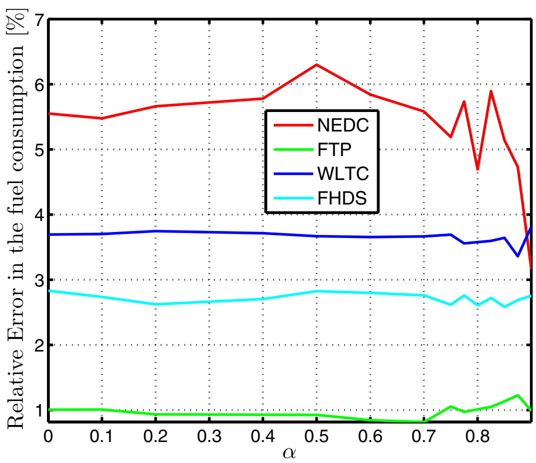

Fig. 6. Relative error in the fuel consumption [%]

From Figures 6 and 7, the maximum error in CO emissions between the strategy (S_2) and $(ECMS_2)$ is less than 5.5% while the maximum error in fuel consumption is less than 6% for the considered driving cycles. This maximum error depends on the driving cycle. The error on the final constraint is illustrated in Figure 8.

To make fair comparisons between (S_2) and ($ECMS_2$), the equivalent fuel consumption defined by

$$Q_{eq} = \frac{1}{1-\alpha}\left[\int_0^T (1-\alpha)c(u,t,\theta_e)dt + \lambda \cdot (\xi(T) - \xi(0))\right]$$

is used. The obtained relative error in Q_{eq} between the strategies (S_2) and ($ECMS_2$) is reported in Figure 9. The maximum relative error in Q_{eq} is less than 2%.

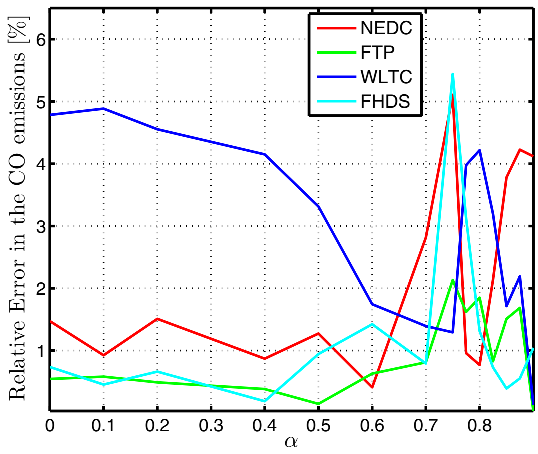

Fig. 7. Relative error in the CO emission [%]

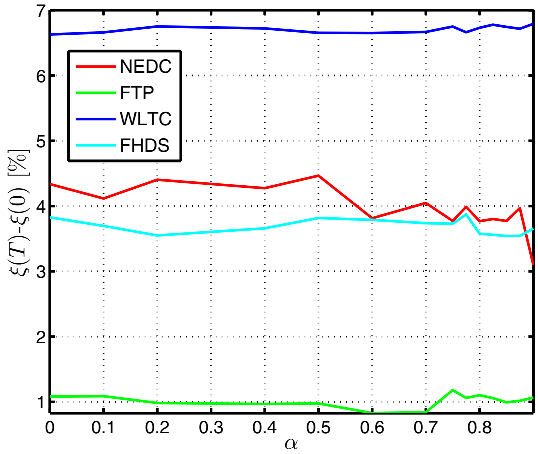

Fig. 8. Final constraint satisfaction

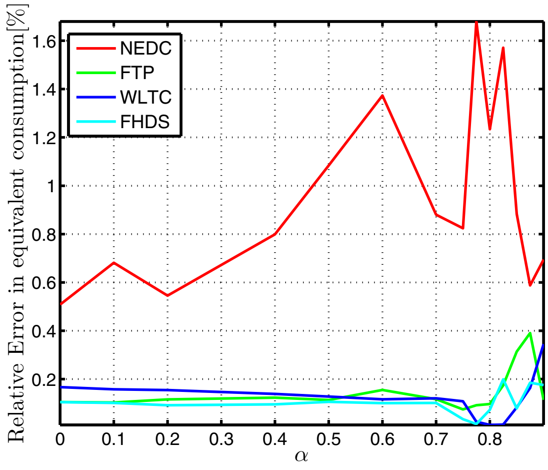

Fig. 9. Relative error in equivalent fuel consumption [%]

To illustrate the added value of the strategies ($ECMS_1$) and ($ECMS_2$), fuel consumption and CO emissions obtained using these two strategies are compared to their optimal values calculated using the strategies (S_1) and (S_2) for cold-start conditions for the NEDC cycle in Figure 10. In this case, the parameters $(s_0,\ k_p)$ are tuned to satisfy the final constraint on the SoC and the full simulation model (3, 1, 4, 5) is used for the comparison.

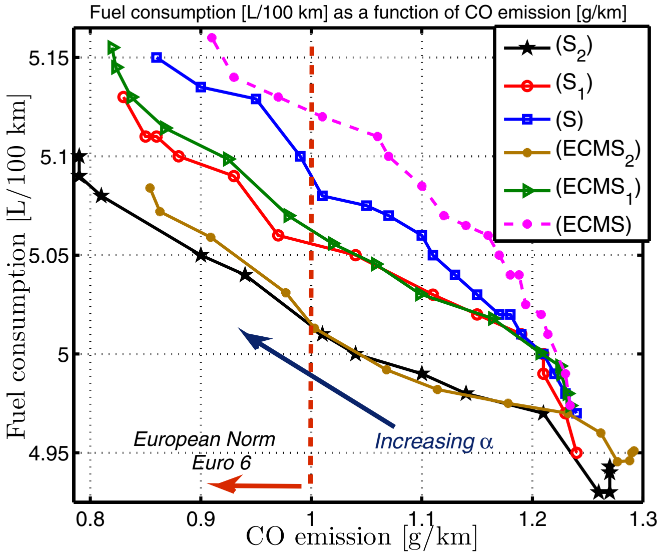

Fig. 10. Fuel consumption [L/100 km] and CO emissions [g/km] for NEDC

These numerical results show that the strategies (ECMS), ($ECMS_1$) and ($ECMS_2$) are close to strategies (S), (S_1) and (S_2), in terms of fuel consumption and CO emissions. These real-time strategies allow the CO emissions to be reduced in order to satisfy the European regulation with an acceptable extra-fuel consumption comparing to the optimal strategy (S_2) calculated with the full knowledge of the driving cycle.

6. CONCLUSION

In this paper, the design of an on-line EMS considering advanced modeling of engine and after-treatment system (3-way catalyst) for a parallel HEV has been addressed. An extension of the ECMS based on correlations between the thermal state and their corresponding adjoint states, observed along extremal has been tested. The proposed correlations have been shown to be robust against the driving conditions change: sub-optimality in the equivalent fuel consumption is less than 2% while sub-optimality in CO emissions is less than 6%. The proposed strategies give satisfactory results for the pollutant emissions reduction with an acceptable extra-fuel consumption compared to optimal strategies.

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Appendix A. RELATION BETWEEN ECMS AND LINEAR QUADRATIC REGULATOR

It is possible to relate the ECMS in its original formulation (considering only the SoC) to the usual linear quadratic regulator (which allows the calculation of an optimal feedback for linear systems and neighboring optimal control in more complex cases. For more details, see (Bryson and Ho, 1969) and chapter 6 of (Maamria, 2015)). To establish this relation, the following assumptions are considered:

- The fuel consumption c_h (for a warm engine) is approximated by a second-order polynomial function in u with the coefficients c_i that depend on the engine rotational speed:
$$c_h(u, \omega_{eng}) = c_0(\omega_{eng})u^2 + c_1(\omega_{eng})u. \qquad \text{(A.1)}$$
- The dynamic of the SoC is approximated by an affine function of the control u:
$$f(u, \omega_{el}) = -\alpha_0(\omega_{el})u - \alpha_1(\omega_{el}). \qquad \text{(A.2)}$$
This assumption is relevant if the power requested by the electric machine is not significant compared to the maximum available power and the losses in the battery are neglected. Otherwise, f is quadratic in u.
- The constraints on the control variable u are neglected, which is equivalent to considering a powerful machine and a large battery.

As the vehicle is assumed to follow a prescribed driving cycle, the parameters $(c_i,\ \alpha_i)$ become functions of time. The cost function to be minimized in the fuel consumption when the engine is warm. It can be obtained from the general cost function in (1) by setting α to zero and by considering a warm engine start ($\theta_e \geq 80°$C). To take into account the final constraint on the SoC, a final quadratic cost is added to the fuel consumption

$$J^m(u) = \beta \cdot (\xi(T) - \xi_{ref})^2 + \int_0^T c_h(u, t)dt. \qquad \text{(A.3)}$$

To satisfy the final constraint (2) exactly, β should be infinite, which is not possible from a numerical viewpoint. Thus, a certain error in $(\xi(T) - \xi_{ref})$ must be allowed. The optimization problem aiming at minimizing (A.3) with the assumptions (A.1, A.2), and in the absence of state and input constraints, is a linear quadratic problem (LQ).

From the PMP, the necessary and sufficient condition for which u_0^* is the optimal control minimizing (A.3) under the dynamics (3), is given by

$$u_0^*(\lambda, t) = \frac{\lambda\alpha_0(t) - c_1(t)}{2c_0(t)}.$$

Applied to this problem, Dynamic Programming gives the costate variable as a function of the state and time. Indeed, $\Im(\xi, t)$ is defined as the optimal cost for the problem (A.3) where the initial time 0 is replaced by $t \in [0, T]$ and the initial state condition is replaced by $\xi(t)$. As the problem is linear quadratic without constraints, its value function $\Im(\xi, t)$, the optimal control and the adjoint state are determined by solving a Riccati differential equation (Bryson and Ho, 1969).

Since the state only appears in the optimal control problem by the difference $\xi(T) - \xi_{ref}$, a variable x defined by

$$x(t) = \xi_{ref} - \xi(t)$$

is used to simplify the calculations. The form of $\Im$ is quadratic with respect to x such that

$$\Im(x^*, t) = \frac{1}{2}v_0(t) \cdot x^{*2} + v_1(t) \cdot x^* + v_2(t).$$

Then, λ^* is given by

$$\lambda^*(t) = \frac{\partial \Im}{\partial \xi}(x^*, t) = v_0(t) \cdot x^*(t) + v_1(t), \qquad \text{(A.4)}$$

where x^* is the optimal state trajectory. In this expression, *λ^* is an affine function of x, with a time varying gain v_0 and a time varying drift v_1*. The parameters v_0 and v_1 are solutions of the time varying differential equations obtained from Riccati equation

$$\frac{dv_0}{dt} = \frac{\alpha_0^2}{2c_0} \cdot v_0^2, \qquad v_0(T) = 2\beta,$$
$$\frac{dv_1}{dt} = \left(\frac{\alpha_0 c_1}{2c_0} - \alpha_1\right) \cdot v_0 + \frac{\alpha_0^2}{2c_0} \cdot v_0 \cdot v_1, \qquad v_1(T) = 0.$$

These equations are solved backward to compute the optimal control, the state and the costate trajectories. Observe that if $\beta = 0$, the parameters $v_0 = v_1 = 0$, which is intuitive since, in the absence of the final constraint, the optimal strategy is to use the electric energy available in the battery as there is no instantaneous constraint on the SoC.

The expression (A.4) of λ as a function of x is similar to the formula (11) in the case $k_i = 0$ where the drift λ_0 and the gain k_p are time varying. This is the analogy one can draw between the single state ECMS and LQ control.