Dynamic optimization of a system with input-dependant time delays

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Abstract

This paper studies one example of dynamic optimisation of systems subject to hydraulic transportation delays. Properly taking into account the variability of the delay in optimisation is a challenging problem, of importance in several applications. While stationary conditions have been derived in earlier works, here we investigate practical numerical aspects and propose a direct resolution method using an orthogonal collocation approach and a state of the art interior point solver. On the basis of the benchmark process considered here, we compute optimal solutions, discuss their surprisingly rich structure and explore the relevance of different numerical schemes.

Keywords

Optimal control, variable time delays, process control.

Introduction

The problem of controlling systems governed by transport phenomena has attracted considerable attention in the recent years. This certainly stems from the fact that these phenomena are ubiquitous in applications, and in process industries in particular: screw extrusion [1], filling dynamics in internal combustion engines [2, 3], crushing-mills [4], multiphasis flows [5, 6], microfluidics [7], blending operations [8], to name a few. When actuators and sensors are not collocated, significant time delays appear in the dynamics. The situation becomes very involved when flow rates are manipulated variables. Then, the delays also become control dependant, with non negligible variations.

Our work is particularly focused on plants where hydraulics are present. Under a plug-flow assumption, the delay in a transport pipe (from the inlet to the outlet) is implicitly defined by the relation [9, 10]

$$\frac{1}{V}\int_{t-D(t,q)}^{t} q(\tau)\,\mathrm{d}\tau = 1 \qquad (1)$$

where q is the flow rate in the pipe (any given branch of the flow sheet of the plant), V its dead volume and D the delay affecting the properties that are being transported and measured at the outlet of the line.

While the incidence of fixed delays on control strategies have been abundantly studied, the cases of time-varying delays or input dependant delays have received much less attention so far. This is so because the closed-loop analysis is much more convoluted, as underlined in the studies by [11, 12, 13, 14, 15, 16, 17, 18, 19], among others. In short, the outcome of these works is that some guarantee can be given to the closed-loop stability of predictor-feedback controllers for such systems. Concerning open-loop design (i.e. design of optimal transient trajectories) the literature does not propose many results for such hydraulic-type varying delay.

Recently, first order necessary optimality conditions of such problems have been studied in [20] but, to the best of the authors' knowledge, little has been done regarding actual numerical resolution of such problems. This is a serious concern in view of real-world applications. In this paper, we aim at bridging this gap between theory and applications. We address the direct numerical resolution of the dynamic optimization of a benchmark problem, a flow rate controlled water heater with downstream measurement. We start

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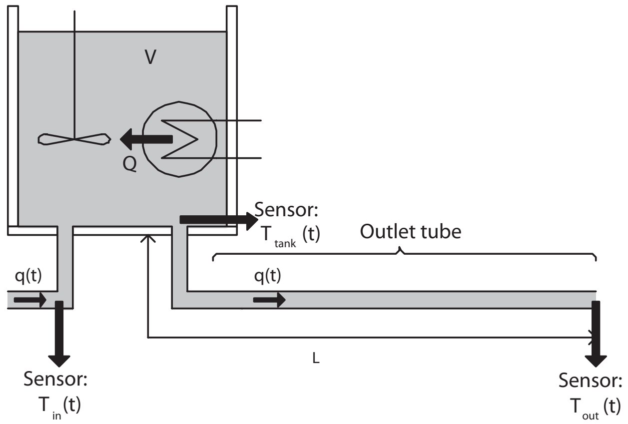

Figure 1. Schematic of the water heating process from [21]

by presenting the example under consideration in this study, then we quickly review the well-known collocation method we seek to apply. Lastly, we discuss the structure of the solutions that arise from the optimisation and the relevance of several numerical schemes. It appears that the surprisingly rich structure of the optimal solution (featuring periodic transient patterns) can only be finely captured with a relatively accurate discretisation scheme, the finite volume method.

Problem statement

The system under consideration here is a flow rate controlled water heater with downstream measurements first introduced in [21] to outline the challenges associated with closed-loop control of systems featuring hydraulic time delays. The system is represented on Fig. 1. It is composed of a tank filled with a constant volume V of water and heated by a fixed thermal flux Q. A controlled flow rate of water q passes through the tank, coming in at a fixed inlet temperature T_{in}. Since water gets heated as it flows through the tank, the outlet temperature of the tank is higher than T_{in}. After having left the tank, water flows through a pipe of (constant) cross-section S over a length L. The variable one seeks to control is the temperature T_{out} at the outlet of this pipe.

Neglecting heat losses, the average temperature in the tank $T_{tank}(t)$ satisfies the following balance equation

$$\begin{aligned}\frac{dT_{tank}(t)}{dt} &= \frac{Q}{\rho c_p V} + \frac{q(t)}{V}(T_{in} - T_{tank}(t)) \\ &\triangleq f(T_{tank}, q)\end{aligned} \quad (2)$$

where ρ, c_p and Q are the density of water, its specific heat and the power of supplied heat, respectively. Assuming instantaneous mixing in the tank, one has

$$T_{out}(t) = T_{tank}(t - D(t, q)),$$

with

$$\int_{t-D(t,q)}^{t} q(\tau)\,\mathrm{d}\tau = LS$$

Given some desired reference signals T_{ref} and q_{ref}, the optimal control problem under consideration in this article is, classically,

$$\begin{aligned}\min_{q, T_{out}} \int_0^T & w_T \cdot \|T_{out}(t) - T_{ref}(t)\|^2 \\ & \quad + w_q \cdot \|q(t) - q_{ref}(t)\|^2 \, \mathrm{d}t \\ s.t. \quad & \dot{x} = f(T_{tank}, q) \\ & T_{out}(t) = T_{tank}(t - D(t, q)), \\ & T_{tank}(t \leq 0) = T_0 \\ & q_{\min} \leq q(t) \leq q_{\max}\end{aligned} \quad (3)$$

In this work, we consider a direct simultaneous resolution approach for (3) using orthogonal collocations as described in [22]. Interestingly, encompassing the input delayed equation in this framework is a challenge, as it results into discontinuous discretized equations. The discontinuity appears in the definition of the indices of the discrete variables. Depending on the values of the discrete unknowns, the indices appearing in the system's equations are changing, implicitly, and discontinuously.

As a consequence, instead of working directly with the delayed equation, a better idea is to replace it with the original transport equation

$$\begin{aligned}&\partial_t T(x, t) = -u(t)\ \partial_x T(x, t), x \in [0, L], u(t) = \frac{q(t)}{S} \\ &T(0, t) = T_{tank}(t), T_{out}(t) = T(L, t)\end{aligned} \quad (4)$$

Formally, this change of representation does not generate any approximation (equation (1) corresponds to the transport lag of (4), exactly).

Numerical treatment of the transport equation

In this section, we describe the numerical methods used to deal with the transport equation (4) before presenting their results, outlining practical numerical difficulties and discussing the interesting features of the optimal solutions.

Finite differences (FD) discretisation

The first straightforward approach we consider is to discretise the transport partial differential equation (PDE) (4) using finite differences (FD) into a set of $N-1$ ODEs, $n = 2...N$

$$\frac{dT_n}{dt} = -q(t)\frac{T_n(t) - T_{n-1}(t)}{\Delta V}, \quad \Delta V = \frac{V_L}{N-1} \qquad (5)$$
$$T_1(t) = T_{tank}(t), T_{out}(t) = T_N(t)$$

Here, collocations are applied on the new set of ODEs including the tank and the discretised transport dynamics (2)-(5), (see [22]).

Finite volumes(FV) discretisation

In a second approach, space is classically divided into a set of cells over which averaged properties are defined (see [23]) (with a running index j)

$$T_j(t) = \frac{1}{\Delta x}\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} T(x,t)\,\mathrm{d}x$$

Then, conservation laws imply that the evolution of the averaged property of each cell only depends on the energy flux at its boundaries, yielding

$$T_j(t_{i+1}) = T_j(t_i) - \frac{\Delta t}{\Delta x}(F_{j+\frac{1}{2}}(t_i) - F_{j-\frac{1}{2}}(t_i))$$

Ultimately, the numerical scheme consists of choosing an appropriate expression of the numerical flux. In this paper, we consider a second order accurate scheme,

$$F_{j+\frac{1}{2}}(t_i) =$$
$$u(t_i)T_{j-1}(t_i) + \frac{1}{2}u(t_i)(1 - \frac{u(t_i)\Delta t}{\Delta x})(T_j(t_i) - T_{j-1}(t_i))$$

It should be remembered that this type of finite volumes numerical schemes is only stable if the Courant-Friedrichs-Lewy [24] condition is verified

$$\frac{u\Delta t}{\Delta x} < 1 \qquad (6)$$

In this approach, the transport PDE is directly transformed into a set of algebraic equations and we only need to apply collocations on the rest of the problem.

Optimization results

All computations were performed using a 2.60 GHz Intel(R) Core(TM) i7-4720HQ processor on a 64 bits system with a 16.0 GB RAM. The discretized optimisation problems were solved using IPOPT 3.11.8 through AMPL. The interior point NLP solver IPOPT is well suited to handle large problems such as those arising in dynamic optimization, see [25].

In the reference case we run our simulations on, we set $Q = 1.10^7$ J.s^{-1}, $V = 1$ m^3, $L = 0.5$ m, $S = 1$ m^2, $w_T = 1.10^3$ and $w_q = 0$. On all our plots, T_{Tank}, T_{out}, ${T_{out}}^{exact}$ and T_{ref} respectively refer to the temperature in the tank, the temperature at the outlet of the pipe as predicted by the optimizer, the real temperature at the outlet of the pipe (computed *a posteriori* from a high fidelity simulator using the optimised input sequence and an explicit characteristics scheme) and the reference being tracked. Finally, n_{fe} and n_{edpd} are the number of elements of the discretization of the time and space domains respectively. The control input is a 0-order hold with time intervals of length $0.1s$.

Numerical results of the optimization for the FD method are reported in Fig. 2-3-4. The case depicted on Fig. 2 exhibits a good match between the transported temperature predicted by the optimizer and its real value during the transient phase, despite some difficulty to encompass the non-smooth points of the transported profile. Around equilibrium, on the other hand, the accuracy of the model is not sufficient to capture high frequency, non-smooth variations, leading the controller to introduce undesirable parasitic oscillations. The settings of Fig. 3 (q_{min} lowered) worsen the situation by leading to a sharper profile that further outlines this limitation of the FD approach, leading to a large prediction mismatch during the transient phase and terrible behaviour around steady state. Finally, Fig. 4 illustrates the same case as Fig. 3 where we have tried to refine the spatial discretization of the PDE to improve the accuracy of the optimiser model. While this comes at a significant computational cost (doubling the discretization leads to a multiplication by a factor close to 8 of the computational load, as expected for a well conditioned optimization problem [26]), the solution is only marginally improved. This shows the limitation of the FD method.

Numerical results of the FV method are given on Fig. 5-6. While Fig. 5 confirms the relatively good results obtained through the FD method in Fig. 2 with the benefit of getting us rid of spurious oscillations, the results of Fig. 6 display a spectacular improvement as compared to the FV approach of Fig. 3-4. On the other hand, we also outline that the computational cost of the FV method is one order of magnitude larger than for the FD one. This is due to the fact that the stability of the numerical scheme imposes the CFL condition (6)

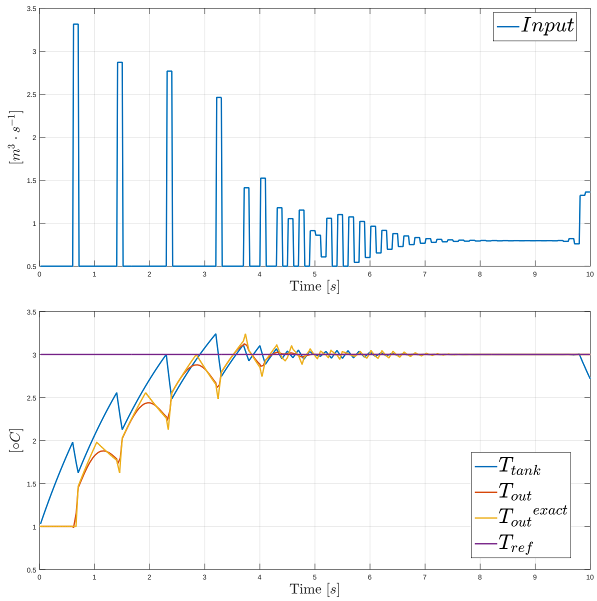

Figure 2. FD discretization, CPU time = 35.08 s n_{fe}=100, n_{edpd}=50, q_{min}=0.5 $m^3.s^{-1}$, q_{max}=5 $m^3.s^{-1}$. [easy case, good performance]

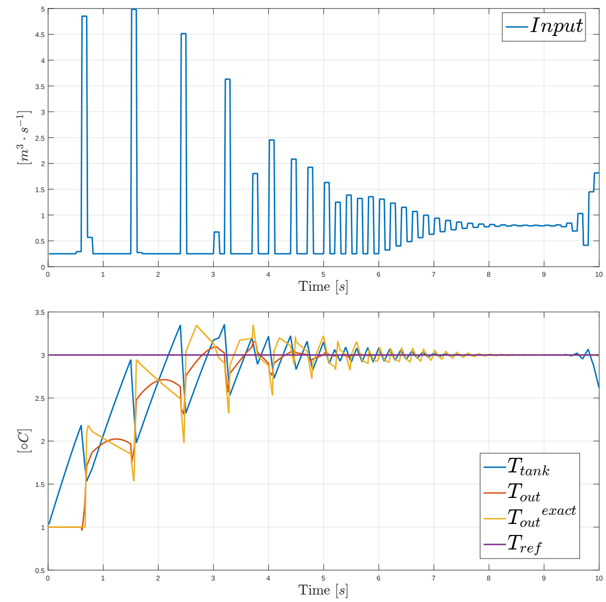

Figure 3. FD discretization, CPU time = 31.841 s n_{fe}=100, n_{edpd}=50, q_{min}=0.25 $m^3.s^{-1}$, q_{max}=5, $m^3.s^{-1}$. [difficult case, poor performance]

and forces us to adopt a very refined time grid to be able to reach acceptable spatial resolution for the transport phenomenon.

Numerical challenges

Because of the time delay, the influence of the control is diminished and problem (3) inherits singular features. As a result the Hessian of the discretized versions of problem (3) is ill-conditioned and remains so as the temporal mesh is refined. The solver, IPOPT, automatically deals with this issue by adding an inertia term to the Hessian to make it full rank (see [25]) but this is known to lead to degraded performances in terms of convergence speed.

An alternative approach to alleviate this issue is to convexify the problem by adding explicitly a penalty term on the control in the objective function. Practically speaking, this means taking $w_q > 0$ (we set $w_q = 50$) in (3). As shown on Fig. 7, when applied to the FD method, this greatly improves the structure of the solution which emulates the behaviour of Fig. 6. This can also be of great practical interest when using the FV method considering that it reduces the computation time by about 45%, as illustrated on Fig. 8.

Another difficulty that we want to outline is that due to condition (6), using an FV approach to discretize the transport equation easily leads to a very large problem. In our example, the order of magnitude of the number

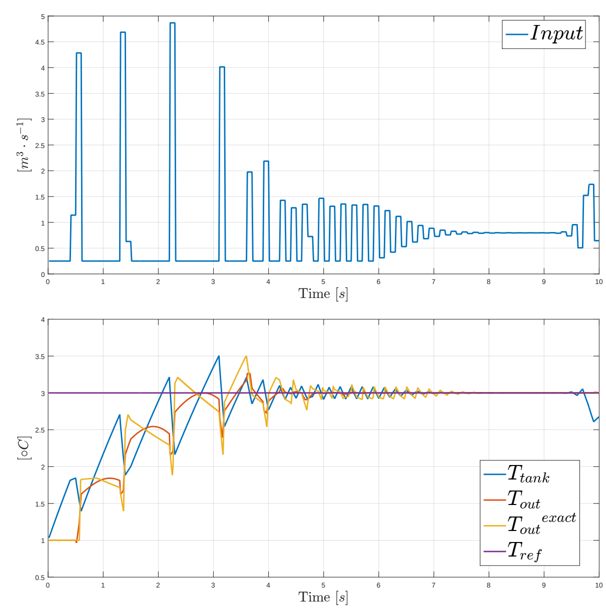

Figure 4. FD discretization, CPU time = 246.053 s n_{fe}=100,n_{edpd}=100,q_{min}=0.250 $m^3.s^{-1}$, q_{max}=5 $m^3.s^{-1}$. [difficult case, poor performance despite mesh refinement]

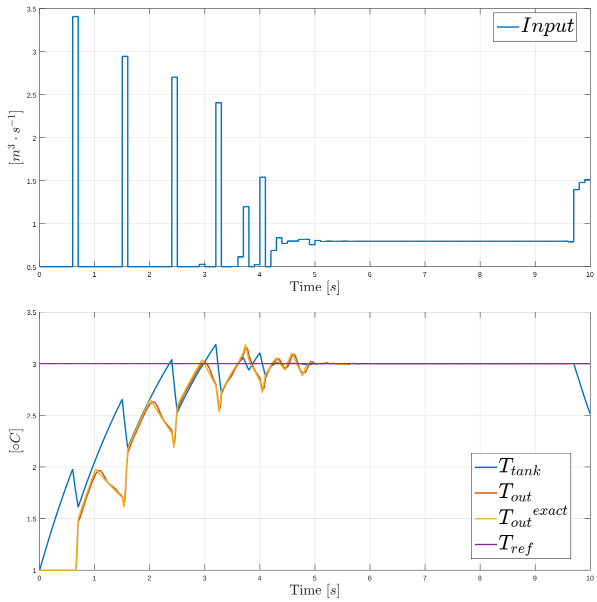

Figure 5. FV discretization, CPU time = 181.403 s n_{fe}=3500, n_{edpd}=50, q_{min}=0.5 $m^3.s^{-1}$, q_{max}=5 $m^3.s^{-1}$. *[easy case, good performance]*

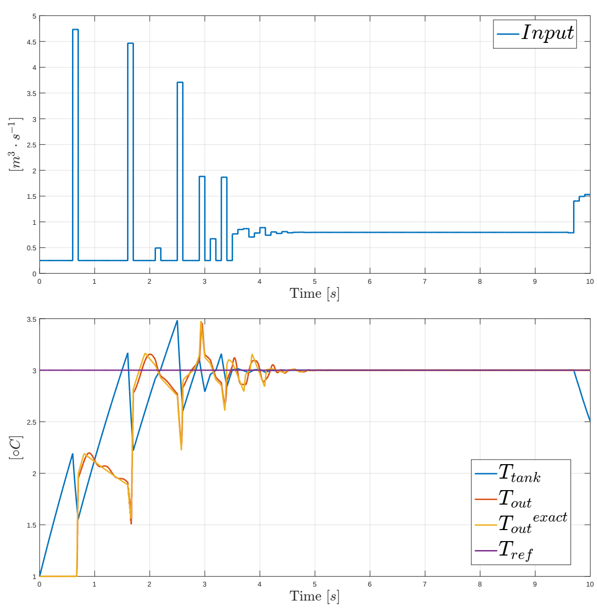

Figure 6. FV discretization, CPU time = 230.045 s n_{fe}=3500,n_{edpd}=50,q_{min}=0.25 $m^3.s^{-1}$, q_{max}=5 $m^3.s^{-1}$. *[difficult case, good performance]*

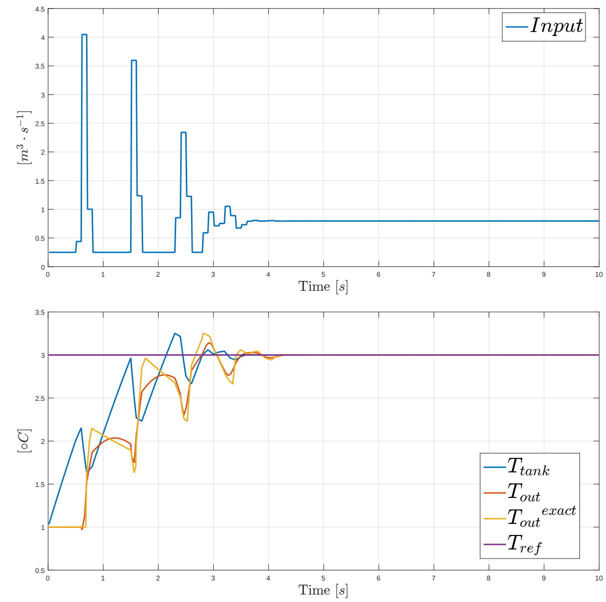

Figure 7. FD discretization, CPU time = 18.202 s n_{fe}=100, n_{edpd}=50, q_{min}=0.25 $m^3.s^{-1}$, q_{max}=5 $m^3.s^{-1}$. *[difficult case, penalty cost, good performance]*

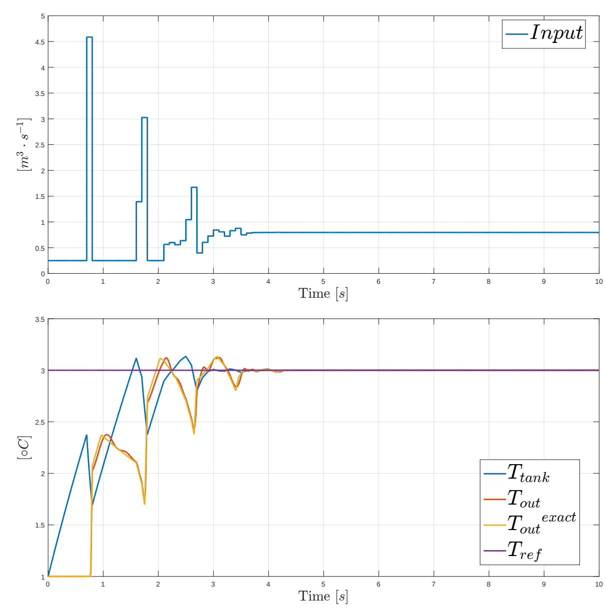

Figure 8. FV discretization, CPU time = 127.591 s n_{fe}=3500,n_{edpd}=50,q_{min}=0.25 $m^3.s^{-1}$,q_{max}=5 $m^3.s^{-1}$. *[difficult case, penalty cost, good performance]*

of variables describing it is 10^5. Besides of the concerns already raised above regarding the associated computational load, this also leads to a large memory use that can prove to be cumbersome.

In the light of previous investigations, we suggest that for applications where performance focus is not too demanding, an FD approach with a suitable control penalization term is probably the easiest way to implement such optimization. If the accuracy achieved this way is not sufficiently good, it may be necessary to switch to an FV approach. This, in turn, will require larger computing resources and possibly the use of some specific strategy such as the advanced step MPC framework presented in [27] to deal with the induced computational delays in a closed-loop implementation.

Structure of the solution

The point that the authors find most intriguing about the various numerical results that have arose throughout this study is the apparently pseudo-periodic structure of the transient state of the optimal solutions computed. Let us denote τ the pseudo-period of the transient part of the input signal.

Despite its complex structure, this type of solution does not necessarily contradict the physical intuition. Indeed, low flow-rate regimes correspond to phases where the water in the tanks gets heated while high flow-rate regimes lead to a quick flush of the outlet pipe (thus allowing hot water to reach faster the outlet of the pipe). This cyclic functioning could hence be described as "heat and flush". A typical example of this structure is presented on the case of Fig. 9, for the purpose of which parameters of the systems have been modified to outline this behaviour ($V = 5m^3$, $Q = 3.10^7 J.s^{-1}$).

In order to gain insight into this phenomenon, we conduct a study of the relation between the value of τ and that of the dimensioning parameters of the problem : V, Q and V_L. The following reference settings are chosen : $V = 1$ m^3, $Q = 1.10^7$ J.s^{-1}, $V_L = 0.5$ m^3, $T_{ref} = 8$ °C. τ is found to be mostly insensitive to V while it exhibits a very structured dependency on Q and V_L, as outlined on Fig. 10-11. It should be noted that while T_{ref} is not considered here as a characteristic parameter of the system, τ still depends on it. As a consequence, the results presented on Fig. 10-11 cannot be directly related to the trajectories of Fig. 2-6 (since optimisation was run with different values of T_{ref}).

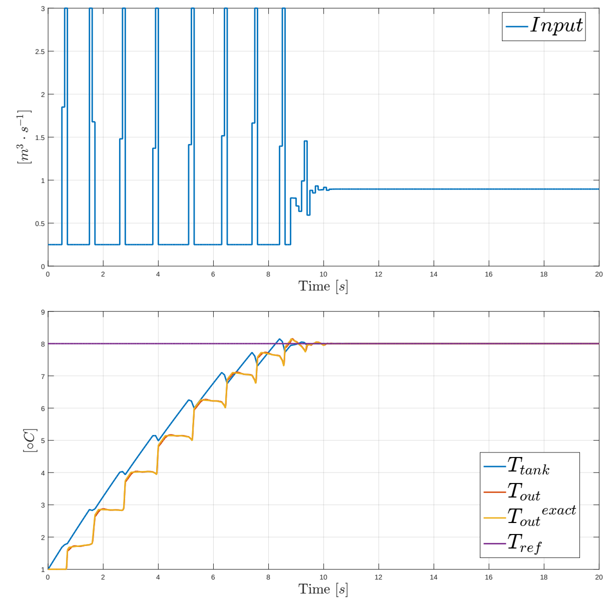

Figure 9. "Heat and flush behaviour", n_{fe}=5000, n_{edpd} = 40, q_{min} = 0.25 $m^3.s^{-1}$, q_{max} = 3 $m^3.s^{-1}$

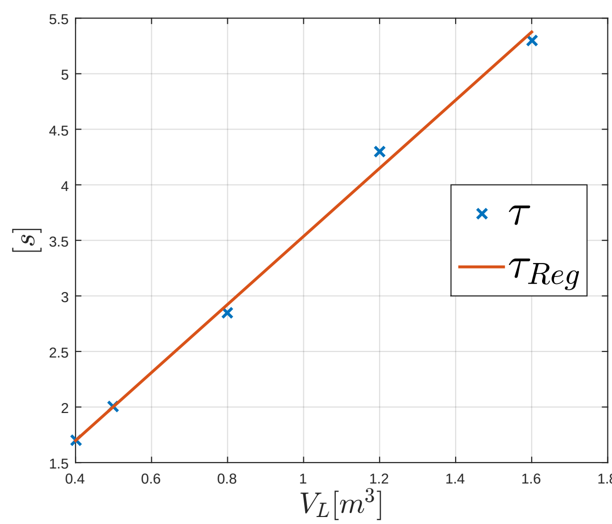

Figure 10. Variation of τ with V_L.

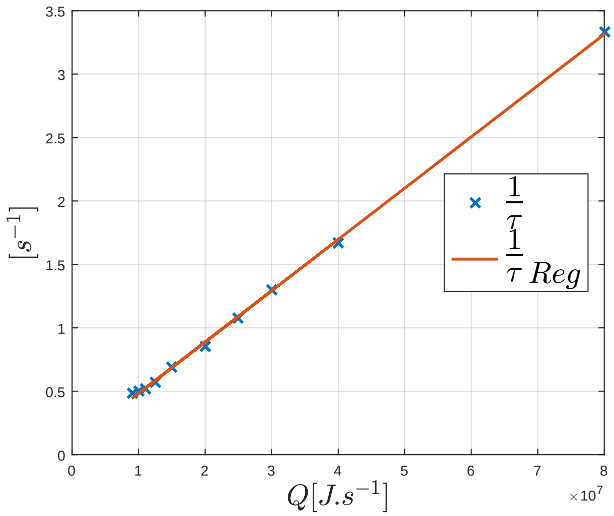

Figure 11. Variation of τ with Q.

Conclusion

In this work, we have presented the results of a practical attempt to compute the optimal trajectory for a system featuring hydraulic time delays. We presented and discussed the relative merits of two alternative numerical schemes balancing computational load versus accuracy. In practical applications, either of them may turn out to be more appropriate depending on the limiting factors of the setting and the focus put on performance.

With that said, we have shown that no matter the approach considered, even for a simple system, taking into account time delays through a transport equation in an optimization problem leads to large dimension systems requiring fairly large computation time. While methods such as advanced-step NMPC could be considered to deal with this difficulty, the authors also believe it would be interesting to investigate the performance of a resolution using an indirect method. Indeed, as shown in [20], first order necessary optimality conditions can be derived directly using the delayed equation solution of the transport PDE, thus allowing to vastly lower the dimension of the problem to solve.

Finally, the optimal solutions have a relatively rich nature, featuring periodic like patterns (here the "heat and flush"). The pseudo-periodic transient's period can easily be related to the value of the various physical parameters of the system. We have not been able so far to provide a theoretical explanation of this phenomenon but will further investigate it in the future.

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