An optimization algorithm for load-shifting of large sets of electric hot water tanks

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Abstract:

Large sets of Electric Hot Water Tanks (EHWT) found in numerous countries appear as very relevant for load-shifting applications. An EHWT heats water over relatively long periods of time, for later consumption. The simple time-of-use pricing strategy applied to most consumers increases the overall electricity consumption in the beginning of the night, while hot water is used much later, in the next daytime. With the development of home automation, more advanced strategies could enable improved cost-reductions for both users and the electricity producer. An efficient and robust improvement is to reschedule each EHWT heating, while keeping the heating periods undivided (to minimize the malicious impacts of complex dynamic effects taking place inside each EHWT), so that an objective load curve for the whole set of EHWT is reached. The first contribution of this article is to formulate the rescheduling of EHWT heating with comfort constraints as an optimization problem. The second contribution is a heuristic to solve this problem with less than 1% optimality loss for representative test cases with large sets of EHWT (typically hundreds of thousands).

Keywords:

Energy storage, Electric water heating, Load shifting, Smart operation, Optimal scheduling, Load curve.

1. Introduction

A key factor in the development of demand side management techniques [1] is the availability of energy storage capacities [2,3]. In this context, the large sets of Electric Hot Water Tanks (EHWT) found in numerous countries appear as very relevant for load-shifting applications and are the subject of various optimization studies. An EHWT heats water over relatively long periods of time, for later consumption [4,5]. While numerous advanced pricing policies have been studied and developed, the time-of-use pricing policy remains of dominant importance for most consumers, e.g. in France. Such policies define broad blocks of hours (for instance on-peak from 6:00 am to 10:00 pm, and off-peak from 10:00 pm to 6:00 am) during which a predetermined fixed rate is applied. The starting times of these periods being known, straightforward heating policies are commonly applied to each individual EHWT which is turned-on immediately after the start of the off-peak period until it is fully heated. This simple strategy increases electricity consumption in the night-time (one period when market electricity prices are low), while hot water is used in the next daytime. At large scales, the result is that the overall consumption of the set of EHWTs rapidly decreases to a low level in the middle of the night, when the electricity production costs are the lowest, unfortunately. This negative effect has to be addressed.

With the development of home automation, more advanced strategies are believed to enable improved cost-reductions for both users and electricity producers [6]. An efficient and robust improvement is to reschedule each EHWT heating in the night time, while keeping the heating period undivided (to minimize the malicious impact of complex dynamic effects taking place inside each EHWT, combining convection, diffusion and stratification [7,8]), to attain a desired load curve for the whole set of EHWTs, while ensuring individual users comfort.

The first contribution of this article is to formulate the rescheduling of a set of EHWT heatings with comfort constraints as an optimization problem. The optimization problem considers hot water consumptions, production objectives and comfort constraints. In view of applications, a large set of EHWTs is considered, each being characterized by its electric power and heat loss coefficient. Initially, a certain heating starting time is assigned to each EHWT having a uniquely defined duration for the subsequent indivisible heating period. Each duration is scaled according to (next day) hot water consumptions. The sum of all the power consumptions defines the initial load curve, as a function of time. The optimization problem that we wish to solve is the rescheduling of each heating to minimize a given criteria (discussed in Section 2). It is the first contribution of the article.

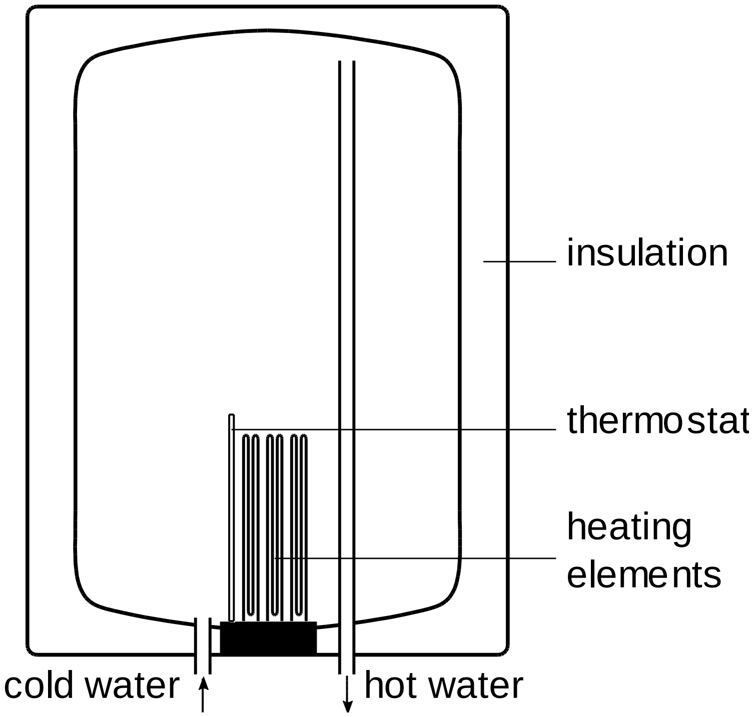


Fig. 1. Schematic view of the cross-section of an electric hot water tank (EHWT)

A heuristic specifically designed for the discrete-time quadratic formulation is proposed to solve this problem. This is the second contribution of the article. The design of any such heuristic strongly depends on the distribution of the durations of the undividable heating. Here, a reference distribution obtained from data measured in a vast set of French households is presented and studied. The data set contains heating periods lasting up to 8 hours. The definition of the proposed heuristic stems from the following considerations. First, one notes that the scheduling of EHWT with long heating durations is the most likely to generate high consumption in on-peak period. Secondly, a rigid individual scheduling for a high number of EHWT is prone to generating singularities resulting in high consumption peaks in the load curve. The heuristic is designed to reduce singularities while generating a high diversity in the distribution. For this purpose, we consider a stochastic heuristic. Its governing principle is to sort the EHWT by decreasing order of duration times, and, then, to randomly schedule them one-at-the-time according to an adaptive distribution law. Each duration times is compensated to account for heat losses. For a given EHWT, this distribution law depends on the residual load curve, which is obtained by constructing the objective load curve minus the power consumption of the tanks already scheduled. To maximize diversity, the distribution law favors scheduling within time periods containing only few previously rescheduled EHWT.

Optimization results produced for real data are given. Objective load curves have been provided by the French utility EDF, and several distribution laws have been tested. They lead to an optimality loss of less than 1%.

The paper is organized as follows. In Section 2, the optimization problem is formulated. Then, in Section 3, the heuristic is presented. Simulation results are reported in Section 4. Conclusions and perspectives are given in Section 5.

2. Formulation of the problem

2.1. Electric water heating

In the following, we consider a set of n electric hot water tanks. An EHWT is a vertical cylindrical tank filled with water [4,5]. A heating element is plunged at the bottom of the tank, and the water is thermally insulated from the ambient using some insulation material, e.g. thermal foam (see Fig. 1). We represent each tank (labelled with $i \in [1, \dots, n]$) using the following parameters and state variables:

- Energy $E^i(t)$ of the water in the tank at time t (defined with respect to cold water). This energy takes value between 0 and a maximum constant E_m^i. It is to be noticed that E_m^i depends on the volume of the tank, and also on a maximal acceptable temperature in the tank (this value is set by the user). In the sequence, the unit used to express energetic contents will be the watt hour (Wh).
- We lump energy losses to the ambient into a heat loss coefficient k^i, representing the heat loss per unit of time (as a percentage of the total energy). The units of k^i is in h^{-1}.
- We note P^i the power of the heating element, expressed in watt (W). We assume that this power cannot be modulated, i.e. that if the tank heating is on, then the injected power is constant and is equal to P^i.

We focus on a time interval $[t_0, t_f]$ in which the tanks are heated, usually during the night. For each tank i, we define a time interval $[t_0^i, t_f^i] \subset [t_0, t_f]$, in which heating of tank i is allowed. For instance, in France, households benefit in the night-time of block of hours in which the electricity price is reduced to promote electricity consumption when the prices are low, and EHWT heating has to be confined to these off-peak periods.

The state-of-the-art heating policy is to switch-on each EHWT at the beginning of its off-peak period until it is fully heated. This strategy leads to an aggregated load curve that rapidly decreases in the middle of the night, when the prices of electricity are the lowest. Depending on the season and the market prices, various load curves can bring substantial savings for the electricity producer (see Fig. 2 where an example of load curve is represented along with market prices during a typical night).

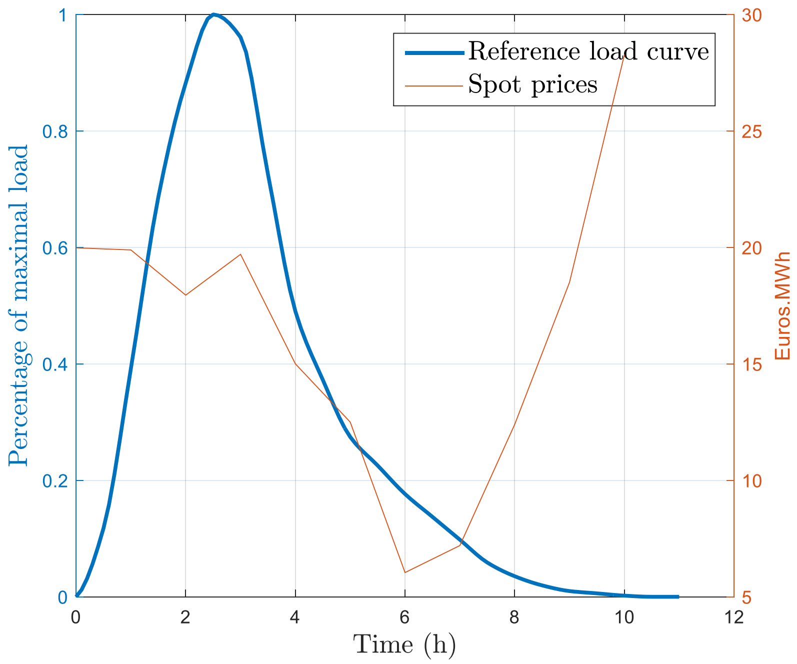


Fig. 2. SPOT market price of electricity (orange) on the night from 2/8/16 to 2/9/16 and typical reference load curve (blue)

The development of home automation opens the way to dependable estimation of the E^i variable and precise setting of the time of heating of each tank. In a general way, we formulate the following problem: given a *reference load curve* $f_a: [t_0, t_f] \to \mathbb{R}_+$ corresponding to the heating of the tanks from the energy $E^i(t_0) = E_0^i$ to $E^i(t_f) = E_f^i$, how can the heating be rescheduled to approach an *objective load curve* $f_b: [t_0, t_f] \to \mathbb{R}_+$?

2.2. Preliminary: Time of heating rescaling and load curve admissibility

Any rescheduling of the heating of any EHWT has to take into account comfort constraints. Due to heat loss, the duration of heating has to depend on the starting time. If the EHWT is heated sooner (respectively later), heat losses are increased and more (respectively less) energy has to be injected in the EHWT. For any EHWT, a change in heating time can be analytically computed as a function of its heating starting time. We address the effect in the following way.

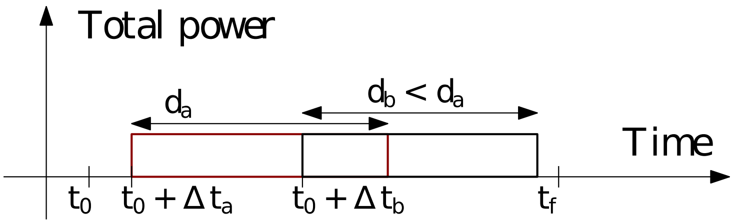


Fig. 3. Heating starting times and durations

For any given tank $i \in [1, \dots, n]$ that, in the so-called reference scenario, starts its heating at time $t_0 + \Delta t_a^i$ for a period of length d_a^i, we wish to estimate heating duration $d^i(\Delta t^i)$ that is needed if one chooses to start the heating at time $t_0 + \Delta t^i$ instead, without altering the energy of the tank obtained at final time t_f (see Fig. 3). A simple integration yields

$$E_f^i = e^{-k^i(t_f - t_0)} E_0^i + \int_{t_0}^{t_f} e^{-k^i(t_f - s)} P^i \mathbb{1}_{[t_0 + \Delta t_a^i, t_0 + \Delta t_a^i + d_a^i]}(s) ds - C^i \quad (1)$$

and

$$E_f^i = e^{-k^i(t_f - t_0)} E_0^i + \int_{t_0}^{t_f} e^{-k^i(t_f - s)} P^i \mathbb{1}_{[t_0 + \Delta t^i, t_0 + \Delta t^i + d^i(\Delta t^i)]}(s) ds - C^i \quad (2)$$

where $\mathbb{1}_{[x,y]}(s) = 1$ if $s \in [x, y]$ and 0 otherwise, and C^i is the total energy consumption on $[t_0, t_f]$ for the tank i. Subtracting (2) to (1) leads to

$$\int_{\Delta t_a^i}^{\Delta t_a^i + d_a^i} e^{k^i s} ds = \int_{\Delta t^i}^{\Delta t^i + d^i(\Delta t^i)} e^{k^i s} ds, \quad (3)$$

and, finally,

$$d^i(\Delta t^i) = d_a^i + \frac{1}{k^i} \ln\left(e^{k^i(\Delta t^i - d_a^i)} + e^{k^i \Delta t_a^i} - e^{k^i(\Delta t_a^i - d_a^i)}\right) - \Delta t^i. \quad (4)$$

Then, if $\Delta t^i > \Delta t_a^i$, we have $d^i(\Delta t^i) < d_a^i$, and if $\Delta t^i < \Delta t_a^i$, we have $d^i(\Delta t^i) > d_a^i$ (see Fig. 3).

This delay effect has an impact on the global load curve. Indeed, to reschedule the time of heating of tanks from a reference load curve f_a to an objective load curve f_b, one needs to ensure that the energy injection is compensated. If we consider that the heat loss coefficients of the tanks are all close to their average k, the same reasoning as before leads to

$$\sum_{i=1}^{n} E_f^i = e^{-k^i(t_f - t_0)} \sum_{i=1}^{n} E_0^i + \int_{t_0}^{t_f} e^{k(t_f - s)} f_u(s) ds - \sum_{i=1}^{n} C^i \quad (5)$$

for $u = a, b$ and then to the following "admissibility" condition for f_b:

$$\int_{t_0}^{t_f} e^{ks}(f_b(s) - f_a(s))ds = 0. \tag{6}$$

2.3. Problem formulation

We can now define our optimization problem. We propose the following formulation. Given n EHWT characterized by their heat loss coefficients k^i, their powers P^i, their allowed time intervals $[t_0^i, t_f^i]$, their initial starting times $t_0 + \Delta t_a^i$ and heating durations d_a^i, define f_a

$$f_a = \sum_{i=1}^{n} P^i \mathbb{1}_{[t_0+\Delta t_a^i, t_0+\Delta t_a^i+d_a^i]} \, . \tag{7}$$

Given an objective load curve f_b verifying (6), we desire to solve

$$\min_{\Delta t_b^1,\dots,\Delta t_b^n} \int_{t_0}^{t_f} \left(\sum_{i=1}^{n} P^i \mathbb{1}_{[t_0+\Delta t_b^i, t_0+\Delta t_b^i+d_b^i]} \, (s) - f_b(s) \right)^2 ds \tag{8}$$

s.t. $\forall i$, the couple $(\Delta t_b^i, d_b^i)$ satisfies $d_b^i = d^i(\Delta t_b^i)$ and $t_0 + \Delta t_b^i + d_b^i \leq t_f$.

3. Resolution method

We propose here a resolution method for (8), with a discretization in p timesteps.

The problem defined above can be related to capacity scheduling problems [9] or cumulative non preemptive scheduling [10]. However the large number of optimization variables, the flexibility yielded by the cumulative nature of power (usually in scheduling problems, the assignment of a job to a machine is required) are not accounted for in classical resolution techniques. Moreover, the problem is also specific due to the time and power scale of the heating. The power consumed in one tank is very small compared to the whole set, because the set it large. By contrast, the duration of the longest heating (up to 7h) is relatively bulky compared to the time horizon (10 to 12h).

In discrete-time, determining whether the problem admits an optimum solution is equivalent to the classical "exact cover problem"[11], which consists in exactly covering a set with a sub-collection of its subsets. Due to the high complexity of solving this problem (shown to be NP-complete in 1972 by R. Karp [12]), and more generally, to the difficulty of minimizing a criteria with a large number of decision variables (the number of tanks can be up to several hundreds of thousands, or millions), we propose in this section an heuristic in the form of a stochastic sequence of rescheduling.

The proposed heuristic is based on the following observations:

- The tanks with long heating durations are more difficult to allocate. On the contrary, short durations yield flexibility to our problem.
- A rigid rescheduling of heating times according to a deterministic procedure is prone to generate singularities in the resulting load curves. Indeed, depending on the shape of the objective load curve, a deterministic rescheduling is very likely to create patterns which generates undesirable high consumptions peaks. On the contrary, due to the large number of tanks, a stochastic heuristic can take advantage of the natural smoothing induced by introducing diversity in the reallocation.

Therefore, we propose the following heuristic. It is based on a *residual load curve* $f_r^i(t)$ which is updated at each step i, and represents the objective load curve minus all powers from the tanks that have been already rescheduled (see Fig. 4). The steps are:

1. Compensate all durations as if tanks where starting heating at t_0, using equation (4).
2. Sort the tanks by decreasing durations.
3. Initialize $f_r^0(t) = f_b(t)$.

4. For all tanks, from $i = 1$ (which heats the longest) to $i = n$ (which heats the shortest), apply the following steps
   a. Using $f_r^{i-1}(t)$, define a set of *admissible starting times* S^i, which is the set among which the starting times can be chosen. In practice, $S^i \subset [t_0^i, t_f^i - d_{t_0}^i]$, and excludes overloaded periods.
   b. Using $f_r^{i-1}(t)$, define a probability distribution law $\mathcal{L}^i$ on S^i that promotes rescheduling in underloaded periods.
   c. Allocate Δt_b^i with respect to $\mathcal{L}^i$.
   d. Update $f_r^i(t) = f_r^{i-1}(t) - P^i \mathbb{1}_{[t_0+\Delta t_b^i, t_0+\Delta t_b^i+d_b^i]}(t)$, s.t. $d_b^i = d^i(\Delta t_b^i)$.

The choice of S^i and $\mathcal{L}^i$ is important. In practice, we give S^i the form of a union of disjoint intervals (in green in Fig. 4 a. and b.), corresponding to starting times Δt^i in which the duration $d^i(\Delta t^i)$ is entirely included in times such that $f_r^{i-1}(t) > 0$ (see Fig. 4 b.). This allows to stop scheduling in already fully loaded periods. Several types of laws can be considered, and have varied efficiency depending on the shape of the objective load curve. If very high slopes are present in the objective load curve, heavy weight should be placed on the boundary of the interval, to promote rescheduling near the boundaries. For this purpose, a parabolic distribution law can be proposed. The law can also directly be defined as a weighted integral of $f_r^{i-1}(t)$ (see Fig. 4 c.).

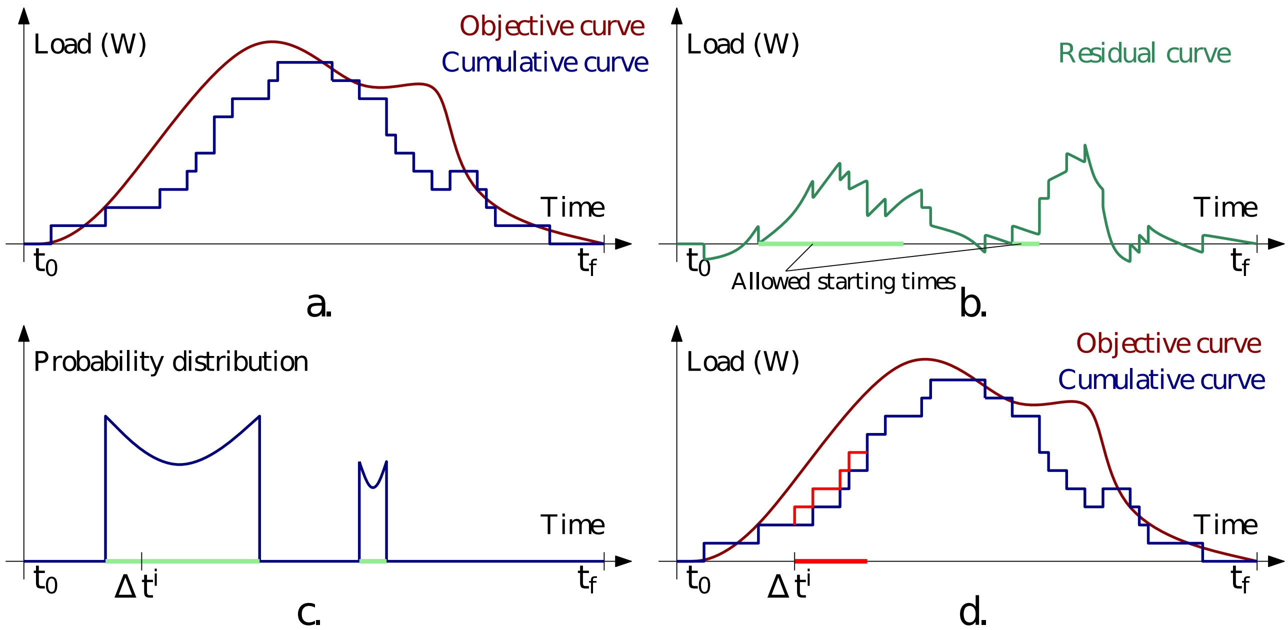


Fig. 4. Detailed process of the heuristic. (a) at the start of an iteration, a cumulative curve has been determined by scheduling some of the heatings. (b) a new heating is to be scheduled, considering its duration, some intervals of possible starting times are determined from the residual curve. (c) along these intervals, probability density functions are defined, they favor the boundaries of the intervals. (d) randomly, a starting time is selected and a new cumulative curve is computed.

4. Simulations results

4.1. Dataset

The feasibility of the problem and the efficiency of the heuristic strongly depend on the shape of the objective curve, the time intervals, and the flexibility yielded by the diversity of the durations among the population of the tanks. For this reason, realistic data have been gathered to test the heuristics.

The distribution of rescaled durations has been constructed based on data gathered on a panel of 267 representative households whose EHWT have been equipped with sensors (see Fig 5). The characteristics of the EHWTs (heat coefficient, power of the heating elements) are taken from representative products on the market. The restrictive time intervals have been defined using the distribution of peak hours in France. The 18 distinct peak hours in France each correspond to a different proportion of the households. Finally, 7 realistic reference objective load curves have been considered.

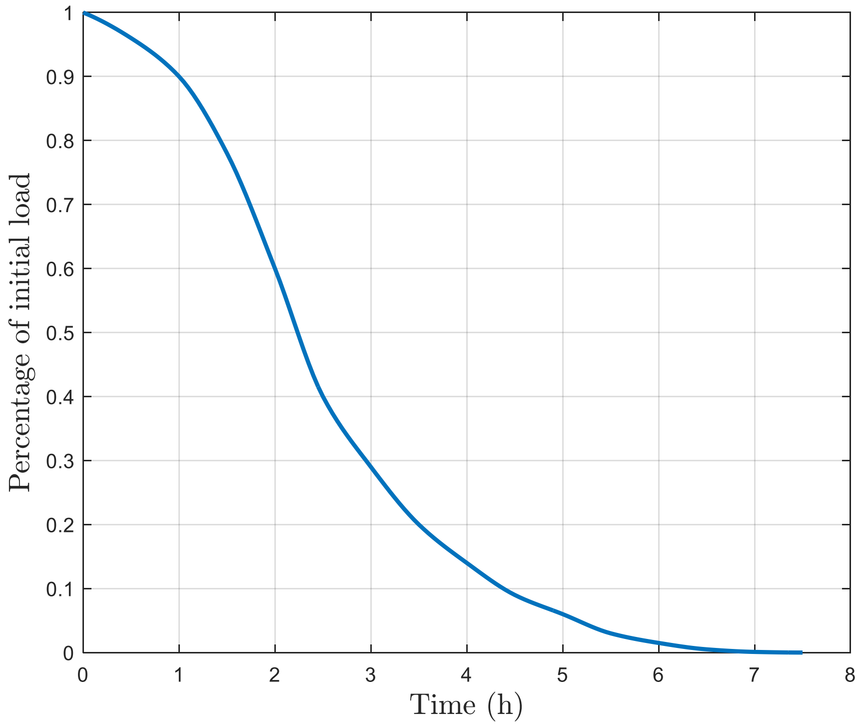
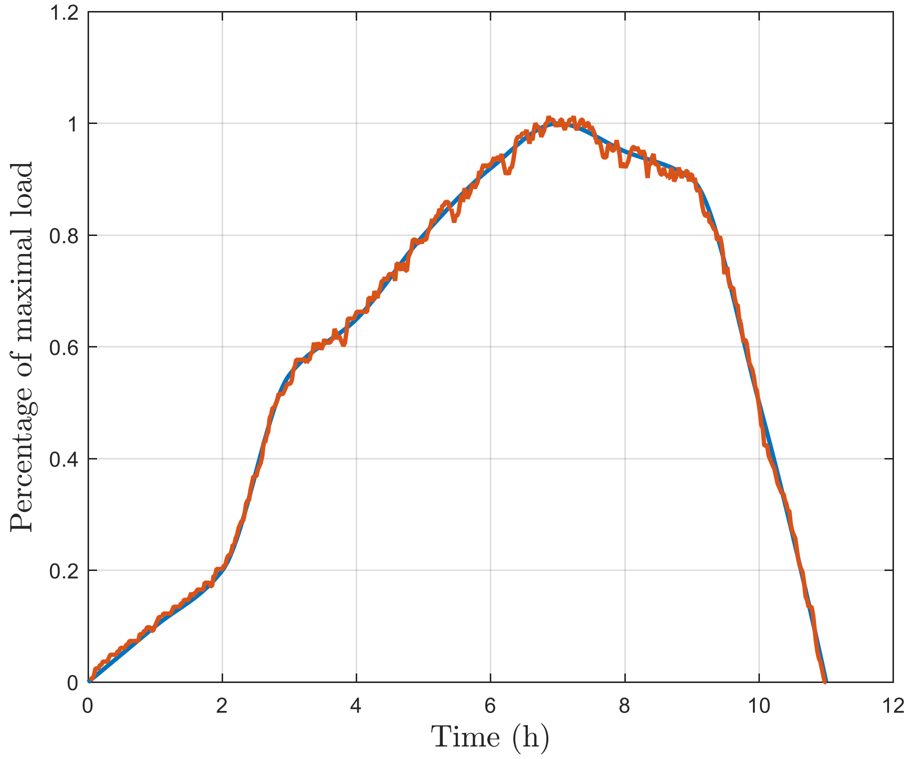


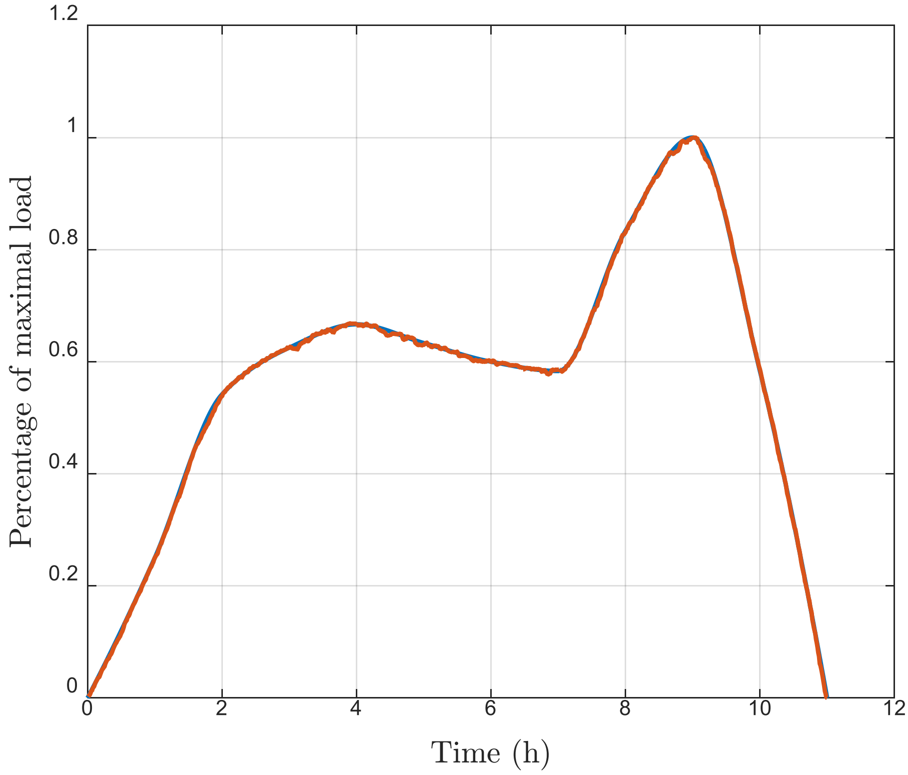
Fig. 5. Typical distribution of durations derived from a representative panel of users, (normalized scale)

4.2. Simulation results

Simulations have been conducted on this dataset. Examples for two different objective load curves are reported in Fig. 6.



a.



b.

Fig. 6. Two examples of objective distribution laws (blue) and final realisations (orange). (a) n=500, p=100 (b) n=5000, p=1000.

To quantify optimality loss, given the obtained load curve $f_o(t)$, we propose two following indices, respectively corresponding to L1 and L2 norms.

$$q_1 = \frac{\int_{t_0}^{t_f} |f_o(s) - f_b(s)|\, ds}{\int_{t_0}^{t_f} |f_b(s)|\, ds}, \qquad q_2 = \frac{\sqrt{\int_{t_0}^{t_f} (f_o(s) - f_b(s))^2\, ds}}{\sqrt{\int_{t_0}^{t_f} (f_b(s))^2\, ds}} \tag{9}$$

Results for the various reference objective load curves (with the best results among various probability law) and various values of n and p are reported in Table 1 and 2. The presented algorithm has been implemented in Matlab R15a, and run on a Intel Core i7 (3,3GHz) with 16GB of RAM, using a single core and 25MB of memory.

Table 1. Results for objective load curve 1

Number of tanks *n*	Number of timesteps *p*	q_1	q_2	Computation time (s)
500	500	0.0166	0.0162	1.8
500	1000	0.0126	0.0122	2.1
500	2000	0.0186	0.0179	4.0
5000	500	0.0032	0.0031	17.6
5000	1000	0.0028	0.0029	22.3
5000	2000	0.0030	0.0032	33.9
50000	500	0.0024	0.0021	131.5
50000	1000	0.0018	0.0017	201.0

The simulations show satisfactory results and highlight the usefulness of favoring diversity during the rescheduling procedure. Indeed, results with a high number of tanks have better optimality index that the one of the small instances. On the contrary, quality of the results is not increasing with the number of timesteps, meaning that discrete time mesh refinement does not bring any significant additional performance.

Interestingly, the proposed heuristic is computed in approx. linear time (with respect to the number of EHWT), and can be applied to very large sets of tanks. However, it is to be noticed that efficiency of a given probability law, and quality of the results depend on the shape of the objective curve and the flexibility yielded by the duration distribution. In our case, with real set and reference objective curve, the flexibility is sufficient.

Table 2. Results for objective load curve 1 to 8

Objective load curve	Number of tanks *n*	Number of timesteps *p*	q_1	q_2	Computation time (s)
1	5000	1000	0.0028	0.0029	22.3
2	5000	1000	0.0036	0.0041	18.6
3	5000	1000	0.0039	0.0042	18.5
4	5000	1000	0.0037	0.0044	17.9
5	5000	1000	0.0032	0.0030	25.6
6	5000	1000	0.0038	0.0042	15.4
7	5000	1000	0.0049	0.0065	17.2

5. Conclusion and perspective

In this paper, we have formulated an optimization problem and a resolution method in the form of a heuristic for the optimal rescheduling of heating of large sets of EHWT. Numerical experiments conducted on real data stress the relevance of this method for parameters corresponding to French houses.

Similar studies should focus on parameters for other countries. Depending on the load curve, which relates to electricity producers constraints and, the heuristic may require further developments. A main tuning parameters is the choice of the probability density employed in the iterative scheduling procedure. In particular, if sharp transients are to be considered in the load curve, the choice of the probability function may require further investigations.

On the numerical side, the approach could benefit from various classic techniques. In fact, parallelization, semi-lumping are possible ways to explore. This could help speed up the method, which is already reasonably fast, but can be important for large instances. A straightforward implementation of the presented methodology treats a representative set of one million EHTWs in 6000 seconds. A satisfactory rescheduling is obtained after 3 random runs.

Nomenclature

Symbols

E energy, Wh

k heat transfer coefficient, s^{-1}

P power injection, W

t_0 beginning time, h

t_f end time, h

Δt heating starting time, h

d heating duration, h

f load curve, W

S set of admissible starting times, h

$\mathcal{L}$ probability law on S

q optimality index

Subscripts and superscripts

i related to tank i

m maximum

a reference scenario

b objective scenario

r residual

Acronyms

EWHT Electric Hot Water Tank

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