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Discrete-time optimal control of electric hot water tank

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Abstract: The paper exposes a discrete time model with three states to represent the dynamics of an Electric Hot Water Tank (EHWT). This models stands halfway between distributed parameters equations and totally lumped single integrators. It allows a faithful reproduction of observed behaviors, especially those induced by stratification. It is also instrumental in formulating optimal control problems aiming at maximizing performance under comfort constraints. In particular, it is shown how to recast such problems as a Mixed-Integer Linear Program (MILP) so that the problem can be solved with off-the-shelf software packages. Numerical results are presented.

Keywords: Integer programming, Linear programming, Energy Storage, Energy Control, Complementarity problems, Discrete-time systems, Dynamic modelling, Load regulation, Optimal control

1. INTRODUCTION

As detailed in numerous studies, the increasing share of intermittent renewable electricity sources (European Commission [2011], Edenhofer et al. [2011]) steadily complicates the management of electricity production-consumption balance. This observation holds both at national and local levels in tension regulation across distribution grids. Demand Side Management (DSM), which is a collection of techniques aiming at modifying consumers' demand, has shown an appealing potential for such concerns (Palensky and Dietrich [2011]). A key factor in the development of DSM is the availability of energy storage capacities. For this reason, network operators and electricity producers are searching and promoting new ways of storing energy. In this context, the large pool of electric hot water tanks (EHWT) found in homes in numerous countries [1] appears as very relevant for load shifting applications, due to its large storage capacity, the flexibility yielded by its geographically scattered characteristic and its functioning.

EHWT are heating water over relatively long periods of time, for later consumption. To reduce cost of operation (from the user viewpoint), a simple but efficient strategy is to use electricity in the night time (a period when electricity price is low), while hot water is used in the next day-time. With the development of home automation, more advanced strategies applied on large pools of EHWT are believed to enable further cost-reductions for both users and utilities. Such strategies can be build on physics-based knowledge of EHWT functioning.

In the literature (Blandin [2010], Kleinbach et al. [1993], Zurigat et al. [1991]), hot water storages are described as vertical cylinders driven by thermo-hydraulic phenomena: *i)* heat diffusion, *ii)* buoyancy effects and induced convection and mixing, *iii)* forced convection induced by draining and associated mixing, and *iv)* heat loss at the walls.

Interestingly, a careful study of the physical principles at stake in the system suggests some simplifications in EHWT modeling. The buoyancy effects lead to the so-called stratification phenomenon (Han et al. [2009]), causing horizontal homogeneity of the temperature. This effect is dominant and allows one to consider only models based on a one-dimensional temperature profile.

Considering that the temperature profile defined above is increasing with height because of stratification, further simplifications are possible. One can define two variables representing the energy contained in the water of temperature above a comfort temperature set by the user (i.e. an energy available for consumption), and a "delay energy". In experiments the temperature increases first at the bottom of the tank where the heating element is plunged, forming a spatially-uniform temperature distribution (which we call *plateau*) which gradually extends itself upwards to the top of the tank. The delay energy represents the energy necessary for the plateau to reach the comfort temperature. Once this level is reached (after the "delay"), further heating raises the energy available

[1] The market share of electric heater is 35% in Canada, see Aguilar et al. [2005], 38% in the U.S, see Ryan et al. [2010], 45% in France, see MSI [2013].

for consumption. A third variable of interest is used to represent the energy in the tank unavailable for consumption (we name it "reserve energy"), i.e. contained in water below the comfort temperature.

This triplet offers a simple representation of an EHWT. Interestingly, the complex evolution of the temperature profile is well represented by the dynamics of this low-dimensional state. This representation is an intermediate solution between partial differential equation models (as in Beeker et al. [2015a] and references therein) and single integrator models. It models the complex dynamics of the comfort variables and the lag observed in the production of hot water and opens the way to optimization. The main contribution of this paper is two-fold: a discrete-time model for the evolution of the triplet, as well as an optimization framework to minimize some criteria (e.g. the cost of heating) and ensure a satisfying supply of hot water for the user.

The paper is organized as follows. Section 2 is dedicated to the definition of the available, delay and reserve energies. Then, in Section 3, we model their dynamics as a discrete-time system controlled by the heating power u. Their dynamics serve to formulate optimization problems. Section 4 shows how to recast such problems as mixed-integer linear programs, which yields easy numerical solution. Numerical results are presented in Section 5.

2. AVAILABLE, DELAY AND RESERVE ENERGY

2.1 General considerations on water tanks and stratification

A typical EHWT is a vertical cylindrical tank filled with water. A heating element is plunged at the bottom end of the tank (see Fig. 1). The heating element is pole-shaped, and relatively lengthy, up to one third of the tank. Cold water is injected at the bottom while hot water is drained from the top at exactly the same flow-rate (under the assumption of pressure equilibrium in the water distribution system). Therefore, the tank is always full. In the tank, layers of water with various temperature coexist (see Fig.2). At rest, these layers are mixed only by heat diffusion which effects are relatively slow compared to the other phenomena (Han et al. [2009]). Existence of a non uniform (increasing with height) quasi-equilibrium temperature profile in the tank is called *stratification* (Dincer and Rosen [2010], Han et al. [2009], Lavan and Thompson [1977]). In practice, this effect is beneficial for the user as hot water available for consumption is naturally stored near the outlet of the EHWT, while the rest of the tank is heated (see Fig.3). Due to this effect and the cylindrical symmetry of the system, one can assume that the water temperature in the tank is homogeneous at each height.

2.2 Modeling assumption

Following the description above, the temperature T of the tank is a continuously increasing function of height (see Fig. 2). We assume that the water injected in the tank is at constant temperature T_{in}, which constitutes a lower bound of the temperature profile, and that the heating process is driven by turbulence generated by buoyancy effects during the heating process, which is the cause of

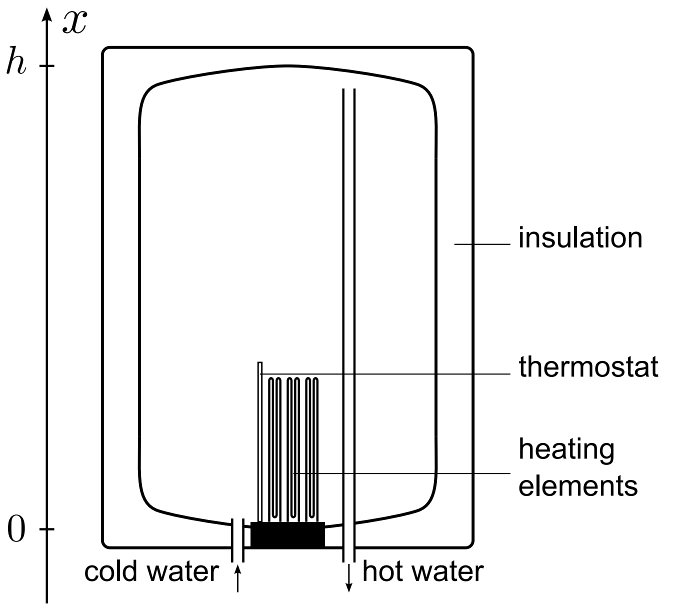

Fig. 1. Schematic view of an electric hot water tank.

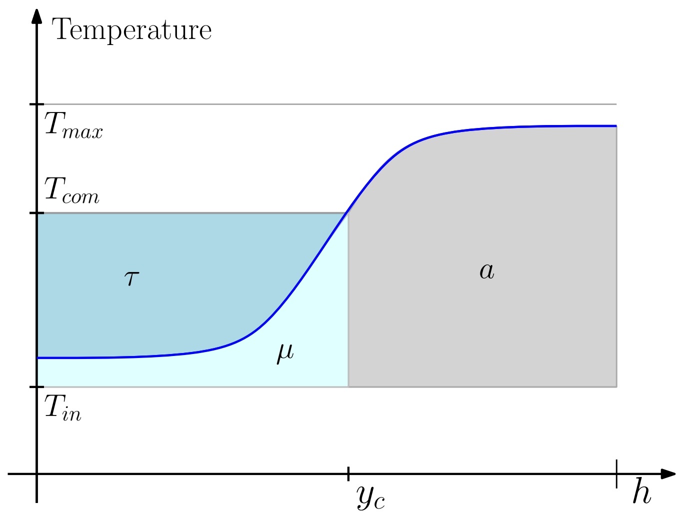

Fig. 2. Temperature profile, and available, delay and reserve energy after a drain.

a local mixing in the bottom of the tank. We consider that this mixing is perfect on a spatial zone, refereed to as the *plateau* (see Beeker et al. [2015a,b]), and does not affect the temperature profile in the upper part of the tank (see Fig. 3). During the process, the plateau grows and gradually covers the whole tank. Thanks to a thermostat, the user specifies a temperature T_{max} at which the heating has to be stopped to prevent overheating. As a result of the heating process, if the temperature at the bottom of the tank is T_{max}, then the temperature in the tank is uniformly at T_{max}.

Moreover, the user can set a comfort temperature T_{com}. Water having temperature higher than T_{com} can be blended with cold water to reach T_{com} and are therefore useful, while water having temperature lower than T_{com} is useless.

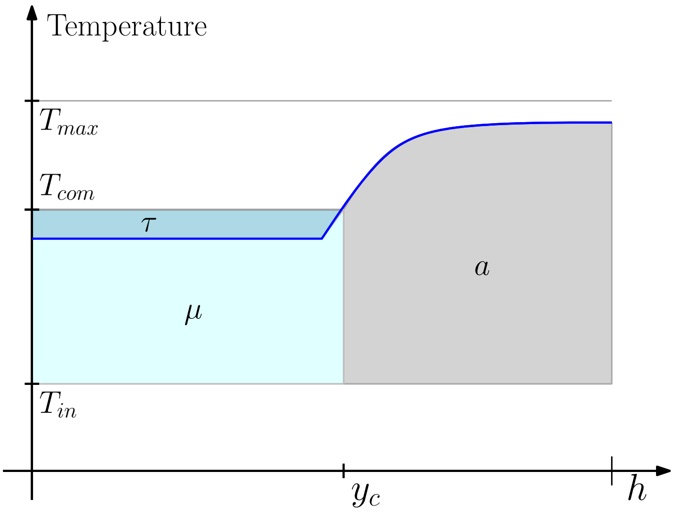

Fig. 3. Temperature profile, and available, delay and reserve energy after some heating.

2.3 Consumption, control and goals

Hot water consumption is an (uncontrolled) input of our problem. For his comfort, the user consumes certain quantities of energy each day. On the other hand, the heat injected via the heating element in the tank is a control variable. The wide development of home automation enable agile piloting of EHWT power injection.

The control can have various objectives. The most obvious one is cost reduction for a single unit in response to a price signal. At larger scales, one can consider a pool of tanks, and try to reach a load profile for the aggregated consumption.

2.4 Variables of interest: Available, delay and reserve energy

Controlling the entire temperature profile inside the tank for optimization purposes appears as unnecessary for the discussed applications. Instead, we simplify the system to a few variables of interest and identify their dynamics. We define the 3 following state variables.

The *available energy* a is defined as the energy contained in the zones having temperature greater than the comfort temperature T_{com}. This constitutes a direct comfort index for the user. If a reaches the value 0 and a water drain is applied, it means that the consumer is trying to consume hot water when none is available, and therefore that the comfort constraints are broken.

The *delay energy* τ is defined as the energy required by the plateau to reach the temperature T_{com}. If the tank is heated at constant maximum power, and without accounting for drains and heat losses, τ is proportional to the time necessary to reach a state in which a can effectively be increased by the heating process.

The *reserve energy* μ is defined as the energy contained in the tank that is currently unavailable for consumption, i.e. the energy contained in the water under T_{com}. When, during the heating process, τ reaches the value 0, the energy μ becomes available to consumption: this generates an immediate (discontinuous) increase of a, and μ is reset to 0.

The rationale behind these definitions is that to plan the heating, we account for the time left before the energy reserve embodied by a (in the total energy $a + \mu$) is consumed, and the time necessary to provide new hot water, embodied by τ.

Fig. 2 and 3 show examples of the dynamics of these variables. A drain (showed in Fig.2) is mainly characterized by a decrease of a and an increase of τ, with a slight raise of μ due to an energy transfer from a. On the other hand, in the heating reported in Fig.3, τ decreases at the same rate as μ rises.

Note h the height of the tank, S its cross-section and $T(\cdot)$ the (non necessarily strictly) increasing temperature profile of the water defined on $[0, h]$. Then, the above definitions yield the following expressions of a, τ, μ as

$$a = S\rho c_p \int_{y_c}^{h} T(y)dy \quad (1)$$

$$\tau = S\rho c_p \int_{0}^{y_c} (T_{com} - T(y))dy \quad (2)$$

$$\mu = S\rho c_p \int_{0}^{y_c} T(y)dy \quad (3)$$

where ρ and c_p are the density and the heat capacity of water, respectively, and y_c is defined as

$$y_c \triangleq \min\{y|T(y) = T_{com}\}. \quad (4)$$

3. DISCRETE-TIME DYNAMICS AND OPTIMIZATION PROBLEM

3.1 Discretization and notation

We consider the variation of $x = (a, \tau, \mu)$ over a finite horizon that we discretize into uniform time-steps $[0, ..., n]$. Note $(a_0, ..., a_n)$, $(\tau_0, ...\tau_n)$ and $(\mu_0, ..., \mu_n)$ the values of a, τ, μ at each of these timesteps.

At each time $t \in [0, ..., n-1]$, energy is consumed by the user via draining by an amount d_t that we consider as known. Energy is introduced via the heating element by an amount $u_t \in [0, u_{max}]$. We divide the later into two parts v_t and w_t, representing respectively the share introduced in a_t and in μ_t. Finally, we define the variable ϕ_t which represents a flow of energy from μ_t to a_{t+1}, taking place in conditions that will be described below.

3.2 Dynamics and constraints

Balance equations For any t, the dynamics of x_t is given by the energy balance

$$\begin{aligned}
a_{t+1} &= (1-p)a_t && - \alpha d_t && + v_t + \phi_t && (A_t)\\
\tau_{t+1} &= \tau_t + p\mu_t && + \beta d_t && - w_t && (B_t)\\
\mu_{t+1} &= (1-p)\mu_t && - (1-\alpha)d_t && + w_t - \phi_t && (C_t)
\end{aligned}$$

The various phenomena described earlier appear in the right hand side of these equations. Heat losses, modeled with an exponential decay, is characterized by the erosion of a and μ at a rate p, the energy from the later contributing to a raise of τ (see Fig. 2). The energy consumed by the user during one time step is split between a and μ with a coefficient α, $1 - \alpha$ and affects τ with a coefficient β.

Definition of the heat source terms v_t, w_t As has been seen before, the energy is injected at the bottom of the tank via the heating element. As a consequence, the heating has no impact on the available energy a when $\tau > 0$, but, instead, tends to reduce τ and increase μ. When the value of τ is 0, the injected energy becomes immediately available. This can be modeled by dividing u_t into two shares v_t and w_t representing, respectively, the part of the injected energy going into a and μ, and subject to the following conditions

$$u_t = v_t + w_t \tag{D_t}$$
$$0 = v_t \tau_{t+1} \tag{E_t}$$
$$0 \leq v_t, w_t \leq u_{max}. \tag{F_t}$$

Given the balance equations (D_t)-(F_t), if $\tau_{t+1} > 0$ then no energy can be introduced in a_{t+1} (i.e. $v_t = 0$ and $w_t = u_t$) , and, if $\tau_{t+1} = 0$ the value of w_t has to compensate for heat losses $p\mu_t$ and/or energy drain from the user βd_t, while the remainder is introduced in a_{t+1}.

Definition of internal energy flow ϕ_t The flow ϕ is always equal to 0, except when τ reaches the value 0. Then, the value of ϕ is defined by the fact that all the energy μ suddenly becomes available. This can be described as follows

$$0 = \phi_t \tau_{t+1} \tag{G_t}$$
$$0 \leq \phi_t. \tag{H_t}$$
$$\tau_{t+1} = 0 \Rightarrow \mu_{t+1} = 0. \tag{I_t}$$

Remark 1. Given a state x_t, a drain d_t and a heat injection u_t, relations (A_t)-(I_t) uniquely define v_t, w_t, ϕ_t and therefore the future state x_{t+1} (under the assumption that τ_{t+1} has to be nonnegative).

Bounds and comfort constraints In this paragraph, we determine some bounds x_t is subject to for all t.

To ensure that the dynamics are properly defined, we impose $\tau_t \geq 0$ for all t.

Moreover, the case where a_t or μ_t is negative corresponds to the drain of non existing energy in the tank and therefore to the breaking of the user's comfort constraints. Therefore, for all t, we require $a_t \geq 0$ and $\mu_t \geq 0$.

Define $m = S\rho c_p(T_{max} - T_{in})$ the maximal energy that can be contained in the tank, and

$$\lambda \triangleq \frac{T_{com} - T_{in}}{T_{max} - T_{in}}.$$

Then necessarily $a_t \leq m$, $\tau_t \leq \lambda m$, and $\mu_t \leq \lambda m$ for all t.

Moreover, physical constraints on the total energy imply

$$\lambda a_t + \tau_t + \mu_t \leq \lambda m \tag{5}$$
$$\lambda m \leq a_t + \tau_t + \mu_t \tag{6}$$

Given these relations, we define the following domain

$$\Omega = \{(a, \tau, \mu) \in \mathbb{R}^3_+ | \lambda m \leq a + \tau + \mu \text{ and } \lambda a + \tau + \mu \leq \lambda m\} \tag{7}$$

Then, $\forall t$, x_t is subject to the constraint

$$x_t \in \Omega. \tag{J_t}$$

This ensures that no energy is drained more that the tank can provide and that the tank is not overheated.

Admissible controls For given $(a_0, \tau_0, \mu_0) \in \Omega$ and $d = (d_0, ..., d_{n-1}) \in \mathbb{R}^n_+$, and for a chosen control sequence $u = (u_0, ..., u_{n-1})$ the relations (A_t)-(I_t) for $t \in [0, ..., n-1]$ uniquely define $(a, \tau, \mu, v, w, \phi)$. This allows us to define the admissible set $\mathcal{U}$:

$$\mathcal{U}(a_0, \tau_0, \mu_0, d) = \{u \in \mathbb{R}^n | \text{the } a, \tau, \mu, v, w, \phi \text{ implicitely defined by } (A_t)\text{-}(I_t)_{t \in [0,...,n-1]} \text{ satisfy } (J_t)\ \forall t \in [0, ..., n]\} \tag{8}$$

In a practical sense, given initial conditions and a drain sequence, it constitutes the set of heating sequences that do not break the comfort constraints of the user.

3.3 Objective function and formulation of an optimal control problem

We consider that a_0, τ_0, μ_0 and d are given. In practice, the electricity producer desires to minimize a given objective function. The most classical example is the cost of heating: given a price signal for electricity over time $(c_0, ..., c_{n-1})$, an optimal control problem can be defined as follows:

$$\min_{u \in \mathcal{U}(a_0, \tau_0, \mu_0, d)} \sum_{t=0}^{n-1} c_t u_t \tag{9}$$

The meaning of this problem is therefore to search, for an EHWT, the heating strategy that minimizes the cost of heating while ensuring a required supply of hot water to the user.

In cases where several tanks can simultaneously be controlled with controls $u^1, ..., u^k$, the cost function can be defined as a quadratic distance to an objective for aggregated consumption $(P_t)_{t \in [0,...,n-1]}$. Then, the problem is

$$\min_{u^1, ..., u^k \in \mathcal{U}(a_0, \tau_0, \mu_0, d)} \sum_{t=0}^{n-1} (P_t - \sum_{j=1}^{k} u_t^j)^2. \tag{10}$$

Generally, the formulation for one or several tanks

$$\min_{u^1, ..., u^k \in \mathcal{U}(a_0, \tau_0, \mu_0, d)} \sum_{t=0}^{n-1} (c_t u_t + \gamma_t (P_t - \sum_{j=1}^{k} u_t^j)^2) \tag{11}$$

with $(\gamma_0, ..., \gamma_{t-1}) \in \mathbb{R}^n_+$, allows to compute the minimum cost for each tank, while approaching some global consumption for the times t with positive γ_t, representing the relative importance of the consumption goal to the price. This formulation can be useful for instance for peak shaving applications.

4. MIXED-INTEGER REPRESENTATION OF THE CONSTRAINTS AND DYNAMICS

If we except the two product conditions (E_t) and (G_t), and the condition (I_t), the relations (A_t)-(J_t) are linear equalities and inequalities in the variables $(u_t, a_t, \tau_t, \mu_t, v_t, w_t, \phi_t)$ and therefore define a polytope of $\mathbb{R}^{7n}$ for each tank. Thus, we adapt (E_t), (G_t) and (I_t) to give to problems (9) and (10) the structure of a linear and a quadratic program, respectively, that can efficiently be solved with commercial software (see e.g. Gurobi Optimization, Inc. [2015] or IBM ILOG [2009]).

Strengthening of (I_t) Given the set

$$A = \{(\tau, \mu) \in [0, \lambda m]^2 | \tau = 0 \Rightarrow \mu = 0\}, \tag{12}$$

then, equivalently,

$$A = [0, \lambda m]^2 \backslash \{(\tau, \mu) \in [0, \lambda m]^2 | \tau = 0, \mu > 0\}. \tag{13}$$

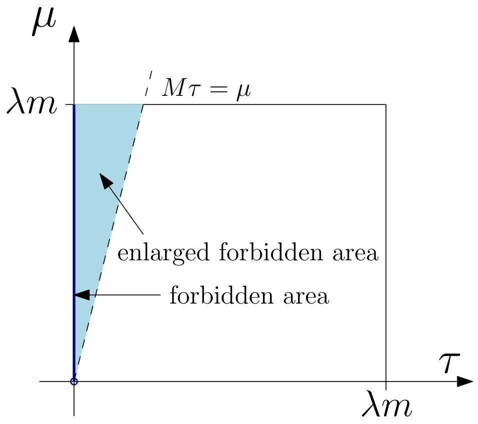

Fig. 4. Strengthening of (I_t).

A possible adaptation is

$$\{(\tau, \mu) \in [0, \lambda m]^2 | M\tau_t \geq \mu_t\} \subset A \quad (14)$$

where $M > 0$ (see Fig. 4). Instead of considering (I_t) we (conservatively) consider (I'_t) which has the linear form

$$M\tau_t \geq \mu_t, \quad (I'_t)$$

chosing $M > 0$ large. This new relation is compatible with a linear programming formulation, and if M is sufficiently large, only a small feasible regime (see Fig. 4) is left out of the optimization problem.

Given this new relation, we define the polytope

$$\begin{aligned}\mathcal{X}(a_0, \tau_0, \mu_0, d) = \{&X = (u, a, \tau, \mu, v, w, \phi) \in \mathbb{R}^{7n} | \\ &(A_t)\text{-}(D_t), (F_t), (H_t), (I'_t)\ \forall t \in [0, ..., n-1] \\ &\text{and } (J_t)\ \forall t \in [0, ..., n] \text{ are satisfied}\}. \quad (15)\end{aligned}$$

Reformulation of (E_t) and (G_t) The product conditions (E_t) and (G_t) correspond to $2n$ sets (v_t, τ_{t+1}) and (v_t, ϕ_t) of two elements that cannot simultaneously be positive. These constraints have the form of *complementarity conditions* (see Cottle and Dantzig [1968]). In practical terms, each of these cases can be encoded with binary or integer constraints and problems (9) and (10) written as a collection of linear programs (LP), respectively quadratic programs (QP), taking the form of a mixed-inter linear program (MILP) and a mixed-integer quadratic program (MIQP).

A more careful look at the situation reveals that these kinds of sets are in fact frequently encountered in discrete optimization under the name of Special Ordered Set (SOS), and are associated with branch and bound strategies (see Beale and Tomlin [1970]). This considerably eases the resolution for instances for reasonable size. These strategies are often implemented in linear programming solvers, see Zuse Institute Berlin [2011].

As a result of the reformulation of (E_t) and (G_t) and the strengthening of (I_t), we can propose the following solution method.

Solution method Given initial conditions (a_0, τ_0, μ_0) and the drain sequence d, solve by any numerical method (e.g. branching strategies) the collection of LP/QP

$$\min_{X^1,...,X^k \in \mathcal{X}(a_0,\tau_0,\mu_0,d)} \sum_{t=0}^{n-1} (c_t u_t + \gamma_t (P_t - \sum_{j=1}^{k} u_t^j)^2) \quad (16)$$

indexed by the $2nk$ SOS on τ and μ.

5. SIMULATION

5.1 Identification of the parameters

In the previously defined problems, values for most of the parameters are easy to determine: T_{com}, T_{in}, T_{max} and therefore λ are either chosen by the user, or directly measured from the water distribution network. On the other hand, the values of m, p and u_{max} depend on the type of tank and are data given by the manufacturer.

The main difficulty is to determine the coefficients (α, β) of our model. They represent the effects of the energy drains $(d_0, ..., d_{n-1})$ on x.

In theory, the values of these two parameters are state-dependent. In practice, their variations are small, except in some parts of the domain Ω, where the temperature in a is close to T_{com} (close to the border $a + \tau + \mu = \lambda m$). Therefore, to keep the linearity of the constraints, we assume that the two parameters are constant, to reduce the cost of this assumption in terms of accuracy, we replace the constraint (5) with $a + \tau + \mu \geq \lambda m + m_p$, with m_p appropriately chosen. Using the model presented in Beeker et al. [2015b] for identification purposes, we set the values $\alpha \simeq 1.2$ and $\beta \simeq 0.4$.

5.2 Simulations results

Exact comparisons of our optimization algorithm with the true optimum for a general model of the tank (for instance a partial differential equations one) is of course impossible, due to the complexity of calculating this optimum. However, to test the relevance of our predictions, simulations with realistic parameters have been conducted on simple cases that can readily be interpreted. The tank parameters correspond to an Atlantis ATLANTIC VMRSEL 200L water tank, and drain histories are taken from the normative sheets emitted by the European norm organism for a tank of such capacity (see European Comittee for Standardization [2010]). The computations were made using the SCIP solver (see Zuse Institute Berlin [2011]).

Several cases have been studied.

Two prices signal In France, most of consumers have two prices for electricity, one for high demand period (mainly in the daytime, for instance from 6:00 A.M. to 10:00 P.M.) and one for low demand period (in the nighttime from 10:00 P.M. to 6:00 A.M., but also for some consumers in between 2:00 P.M and 5:00 P.M.), to increase consumer's sensitivity to the price of electricity.

The majority of already installed tanks receive a wired communication signal of these prices, and start at night, when the low price period begins. However, most of them are completely heated in the middle of the night, which causes a decrease of the demand at the end of the night. This is both harmful for energy producers and not energy efficient, as heat losses take place in a longer period.

In Fig. 5, we report the values of the control and the associated available water for problem (9) with those prices. The optimization has been computed in a 48h

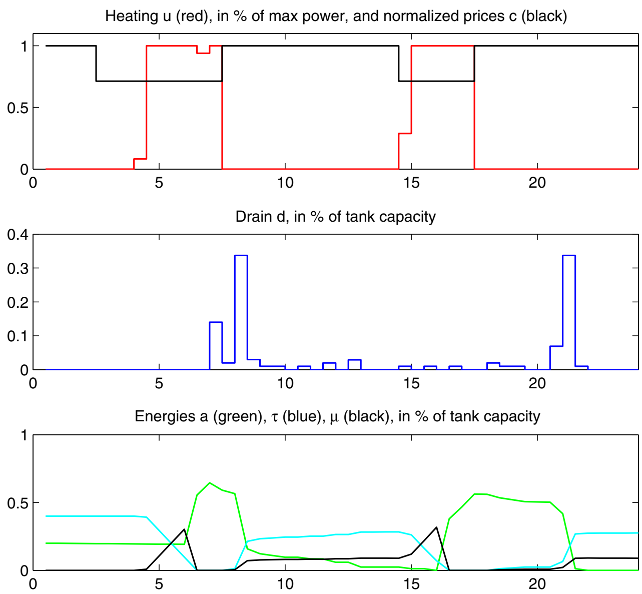

Fig. 5. Optimization for a a two price scenario.

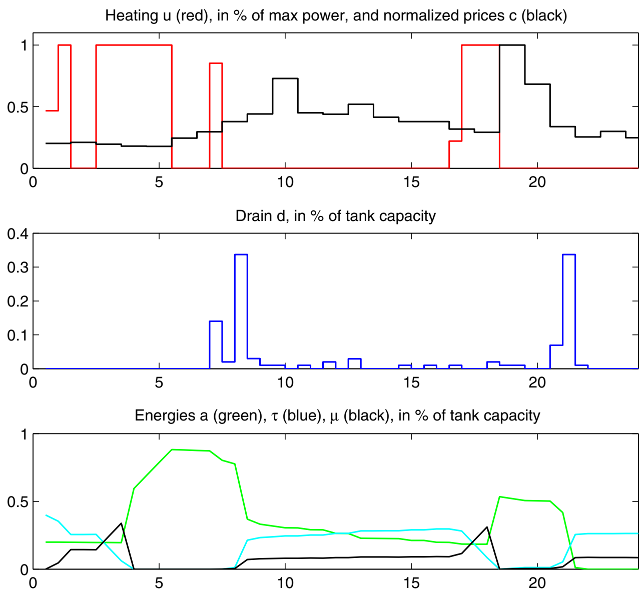

Fig. 6. Optimization for a spot prices scenario.

horizon for a 24h application of the control signal, to avoid side effects. The results show a reasonably varying control signal, which heats the tank at the end of low prices period, to limit consumption at high prices and reduce heat loss. This strategy is more efficient than the one usually set up in practice.

Spot prices In Fig. 6, we report the values of the control and the associated available water for problem (11) with the spot prices in Europe for a typical winter day, taken from the EPEX Spot Market (see EPEX Spot [2013]). Again, the results show a sensible control, which heats the tank at the end of low prices period, to limit consumption at high prices and reduce heat loss.

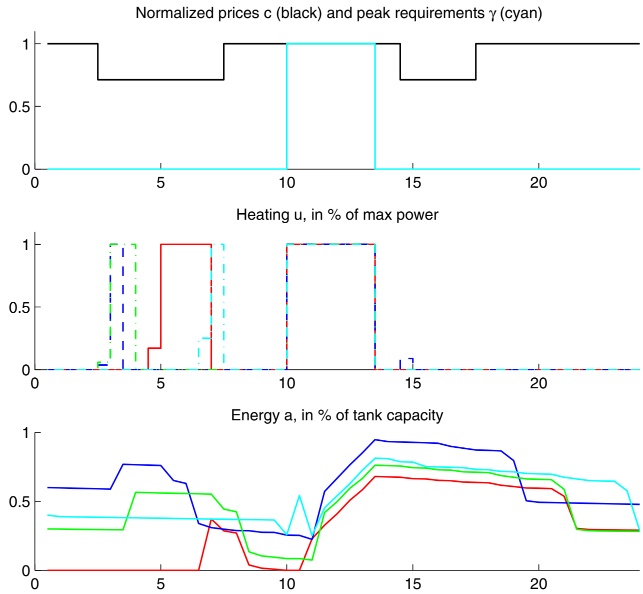

Fig. 7. Simultaneous optimization of four tanks.

Peak shaving for several tanks Finally, we ran simulations on a set of several tanks with various initial conditions, for the problem (11): each tank is optimized against a price signal, and a global consumption is encouraged in the middle of the day. The results of these simulations are depicted in Fig. 7, where each tank represented with a different color. It is to be noted that while each tank has its own optimal strategy, these coincides in the $[10h, 13h]$ time frame where the quadratic incentive has been designed.

6. CONCLUSION AND PERSPECTIVES

This paper proposes a simple 3 states dynamics of an EHWT and a framework to optimally heat it with off-the-shelf solvers. However, practical online computation of heating strategies should account for the inherent stochasticity of users' draining. We have not looked into this problem yet. The definition of a set of realistic scenarios of drains for a given household, as well as stochastic programming appear as promising solution.

Also, some sort of closed-loop feedback model predictive control should be employed. Heating strategy could be computed on a 48h horizon, with a 15min closed-loop interval.

Finally, the generalization to an arbitrary number of tanks requires additional investigation. The computational cost seems to increase exponentially with the number of tanks (see Table 1). Thus, a global piloting using a single optimization program does not appear as a viable solution. However, the tanks having separable constraints,

Table 1. Computationnal times

Number of tanks	Time
1	2.0s
2	9.4s
3	754.2s
4	4756.4s

the optimization problem under consideration is compliant with decomposition-coordination techniques. Further work focus on using the so-called price coordination (see Mesarovic et al. [1971]) with the presented single tank linear programs.

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