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Systems with uncertain and variable delays in the oil industry: some examples and first solutions

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Abstract: In this article we expose typical examples of systems from the oil industry having variable delays. The root causes of the variability can be the transport phenomena, the clocks mis-synchronisation in the employed information technology, or the transmission of waves in surrounding medium. We discuss these problems and sketch solutions.

Keywords: process control, uncertain delays, variable delays, monitoring algorithms, synchronisation of data, information technology, feedback control

1. INTRODUCTION

As many industries, the oil sector has to cope with dynamical systems with delays. One main reason why delays are ubiquitous in this field is that transport of fluid material is a dominant problem in almost all applications related to oil. Another factor is the long distances over which fluid transport (horizontal or vertical) has to be considered. In this article, we present several representative practical examples. With simplifying assumptions, we expose some problems where mitigation of the effects of delays are the central question.

Control engineers know that delays have negative impacts on closed-loop control. However, the malicious effects of the variability of the delay are often underestimated. The examples chosen in this article all feature varying delays. We explain why, and we stress why this is a problem. After some recall on recent methodological tools developed to control delay systems, the paper covers three distinct types of variation.

First, we explain the *control-induced delay variations*. In the blending problem we consider, the delay is defined by an implicit integral equation where the controls, which are flow-rates, have an effect. In this case of deterministic variations, we stress the surprising non-symmetric behavior observed during step-ups and step-downs responses. The predictability of the delay allows one to compensate for it with good accuracy, using a motion planning technique for open-loop and a generalized predictor for closed-loop. Interestingly, this poses challenging stability analysis problems, and we sketch solution for them.

Second, we explain the problems associated with delays caused by *mis-synchronization of data* produced by geographically distributed instrumentations. Here, the delay is uncertain and can not be compensated for. We stress its harmful effects on a simple, but state-of-the-art, monitoring algorithm employed to check the mass balance of an oil and gas production network. As will appear, delay induced by dating uncertainty can be more detrimental than measurement noises.

Third, we consider the problem associated with *non-causality of communication over networks.* Inside a vertical well, we expose how the system of communication with repeaters can cause misinterpretation of measurements when received at the surface. This problem lies at the frontiers of our investigations. We briefly discuss how to address it.

2. NEW CONTROL METHOD FOR DELAY SYSTEMS

The techniques of delay compensation are not new. The most widely used methods are predictor approaches (see e.g. in Artstein (1982); Kwon and Pearson (1980); Manitius and Olbrot (1979); Smith (1958)). As established in numerous surveys and research works (Niculescu (2001); Richard (2003)), the lack of robustness of this technique with respect to the uncertainty on the delay is still a major concern in automatic control theory. This lack of robustness often appears as a performance bottleneck in applications (see e.g. Mondie and Michiels (2003)).

Lately (see Krstic (2008, 2009b); Krstic and Bresch-Pietri (2009); Krstic and Smyshlyaev (2008)), a new class of predictor-based techniques has been proposed to address this uncertainty. In particular, this methodology is based on the seminal idea (see e.g. Krstic (2008)) of modeling the actuator delay as a (fictitious) transport partial differential equation (PDE). Essentially, this is an analysis tool, useful to establish convergence. In details, one uses a backstepping boundary control method on the transport PDE introduced to model the delay. This transformation allows to use systematic Lyapunov design tools for robust stabilization and adaptation. A list of references on this topic includes Bekiaris-Liberis (2014); Bekiaris-Liberis and Krstic (2013a,c); Krstic (2009a); Bresch-Pietri et al. (2014, 2012a,b,c). We now sketch a (brief and partial) state-of-the-art in relation to the examples presented in this article.

2.1 Exact compensation of a single delay

Consider the following system

$$\dot{X}(t) = AX(t) + BU(t - D)$$

where D is a constant delay. Due to the delay, the system is infinite-dimensional. When the delay is compensated, the system becomes finite dimensional, because it becomes delay-less. For a constant delay, exact compensation can be achieved by using a finite time prediction over the exact value of the delay Artstein (1982); Kwon and Pearson (1980); Manitius and Olbrot (1979), i.e.

$$\begin{aligned} U(t) &= KX(t + D) \\ &= K\left[e^{AD}X(t) + \int_{t-D}^{t} e^{A(t-s)}BU(s)ds\right] \end{aligned} \quad (1)$$

where the feedback gain K stabilizes the delay-free dynamics. This is, as is well-known, a non robust control strategy. In particular, uncertainties in the system dynamics and delay reveal troublesome. Fortunately, some degree of robustness can be added by employing adaptive control techniques developed on the basis of this prediction technique. For example, one can refer to Bresch-Pietri et al. (2012a) where several classic cases of equilibrium regulation are treated: parametric uncertainties, disturbance rejection, partial state measurement, or delay adaptation.

2.2 Robust compensation of a single varying delay.

Following Krstic (2009a), consider the more general system

$$\dot{X}(t) = AX(t) + BU(t - D(t))$$

In the case of a varying delay, the prediction has to be done over a varying horizon. This gives (see Nihtila (1991))

$$U(t) = KX(\eta^{-1}(t)) \quad \text{where} \quad \eta(t) = t - D(t) \qquad (2)$$

Importantly, for this controller to be well-defined, the η function has to be invertible (as one has to use its inverse η^{-1}). This means that every information sent has to be received once and only one by the system. A sufficient condition for this is

$$\dot{D}(t) < 1, \quad \forall t \qquad (3)$$

which we will refer from now-on as "causality condition".

In general, the prediction formula (2) does not provide exact delay compensation, since future variations of the delay are not known in advance. We have

$$\dot{X}(t) = AX(t) + BX(t \underbrace{-D(t) + D(t - D(t))}_{\neq 0})$$

At least, one shall investigate the possible impact of this mismatch on asymptotic stability. This can be done by studying a partial differential equation reformulation using a special backstepping transform. This rewriting allows a Lyapunov-Krasovskii analysis. A result is that if the control gain K in (2) can be chosen sufficiently small, then the closed-loop system is asymptotically stable (Bresch-Pietri et al., 2014, Theorem 1) [1].

2.3 Non causal delay

The generalized predictors (2) and their extensions have the capability of treating variable delays and uncertain delays (Bekiaris-Liberis and Krstic (2013a,b); Krstic (2009a)). However, all these works share the common assumption (3).

If this assumption fails, then the principle of causality is violated. The delay increases faster than the time grows. Under such circumstances, information transmitted through a channel delayed in this way does not constitute a continuous flow of data, but produces an intermittent flow. Also, the rule of first-in first-out (FIFO) does not hold anymore.

Assumption (3) has been instrumental in all the works conducted so far. It has appeared both explicitly or implicitly, as a consequence on bounds formulated in the statements of convergence results.

Interestingly, temporary violation of this assumption is not necessarily causing major trouble in the stability analysis. It is more a condition that shall be satisfied "on average", as has been formulated in Bresch-Pietri and Petit (2014), under the relaxed form

$$\frac{1}{t - h_i}\int_{h_i}^{t} \dot{D}(\tau)^2 d\tau < \delta, \quad \forall t \in [h_i, h_{i+1}] \qquad (4)$$

for some ordered sequence (h_i) of discontinuity points $\lim h_i = +\infty$, $\underline{\Delta} \leq h_{i+1} - h_i \leq \overline{\Delta}$. Of course, (3) implies (4).

3. CONTROL-INDUCED DELAY VARIATIONS: TRANSPORT PHENOMENA

We now present a first example where the delay depends on past values of the control. Consider a transport phenomena where the control variable is, directly, or indirectly, the flow-rate [2]. Consider that the flow is incompressible, single dimensional, so that the flow-rate is (spatially) uniform but time-varying. At any instant, the flow-rate can be freely changed (within some physical upper and lower limits). However, propagation of material takes time. If the nature (e.g. concentration) of the fluid matters, then a delay appears, as a simple effect of finite-speed propagation of medium. This is the case in flow networks employed for blending semi-finished products in refineries. This example is pictured in Figure 1.

The flow discussed above satisfies a simple conservation principle (leaving out the effects of viscosity), which is equivalently written under the form of a simple partial differential equation defined over a spatial domain $x \in [0, 1]$

$$\partial_t \xi(x, t) = u(t)\partial_x \xi(x, t)$$

where ξ is the propagated state and u in the input (flow-rate). The (smooth [3]) solutions of this PDE are such that

$$\xi(1, t) = \xi(0, t - D(t)) \qquad (5)$$

where $D(t)$ is defined by

$$\int_{t-D(t)}^{t} u(\tau)d\tau = 1 \qquad (6)$$

Using (5), one defines a delayed input-output relation. This delay is defined by the implicit integral equation (6).

[1] This result has (indirect) connections with the usual robustness margin determined from the Nyquist criterion for LTI systems

[2] It may be necessary to clarify that the flow-rate can be itself a distributed variable, as is the case of compressible flow Di Meglio et al. (2012a,b); Sinègre et al. (2005)

[3] Implicitly, we ignore shocks.

By propagating this reasoning from downstream sections of the networks to upstream (see Figure 2), we define a cascade of input-output relations. Eventually, the control also appears in the definition of each blending, more precisely, in the ratio of the blending. Ignoring the delay dependence on the control, the system is a linear MIMO system. With the dependance of the delays, it becomes a very intricate system.

Generally, systems having a delay of the form (6) are difficult to control and present a non-symmetric behavior for step-ups and step-downs. We report here results from Barraud (2006) in Figure 3, and Figure 4 for a first-order linear system with an input delay of the form (6). The presented results were obtained using a Smith predictor, with a low-pass filter in its outer loop bringing a valuable level of robustness.

However, the complex dependance of the outputs of the system with respect to the inputs can be inverted. As is shown in Petit et al. (1998), it is possible to completely determine the input histories corresponding to any output histories, provided that the number of outputs equals the number of inputs. The problem of motion planning can thus be solved, at least formally.

To understand the basics of the algebraic method employed to parameterize all the solutions of the system [4], we simply recall that in each section (say the i^{th}) of the network, the delay is defined by

$$\int_{t-\Delta(t)_i}^{t} u_i(\tau)d\tau = V_i$$

where V_i is a liquid holdup (volume).

Therefore, by introducing the output

$$y_i(t) = \int_0^t u_i(\tau)d\tau$$

we deduce the delay histories as

$$\eta_i(t) \triangleq t - \Delta(t)_i = y_i^{-1} \circ (y_i(t) - V_i)$$

(where $\circ$ denotes the function composition) and then

$$\eta_i^{-1}(t) = y_i^{-1} \circ (y_i(t) + V_i)$$

This property of the system is instrumental for solving two important problems: planning the production for sharp transient, and reconciling data.

In the first of these two problems, it is necessary to determine the relevant control histories to produce a given amount of some finished product, followed by another amount of a different product, and so on in a sequence defined according to the market demand and the availability of semi-finished products. Each finished product corresponds to a very different recipe of semi-finished products. In turn, this defines very different set of values for the delays appearing in the various parts of the network. It shall be noted that the property of invertibility discussed above implies that, formally, any smooth transient between outputs can be achieved thanks to the inputs. Therefore, in finite time, the production can be achieved without any product losses that would be caused to off-specs production. This problem can be solved for any network admitting a binary tree representation. The algorithm permitting to reconstruct the input histories is as follows, given here for a 3 inputs network with 1 pre-blend (see Figure 2) and 2 liquid holdups (volumes) v_1, v_2.

[4] which is said to be δ-flat Mounier (1995)

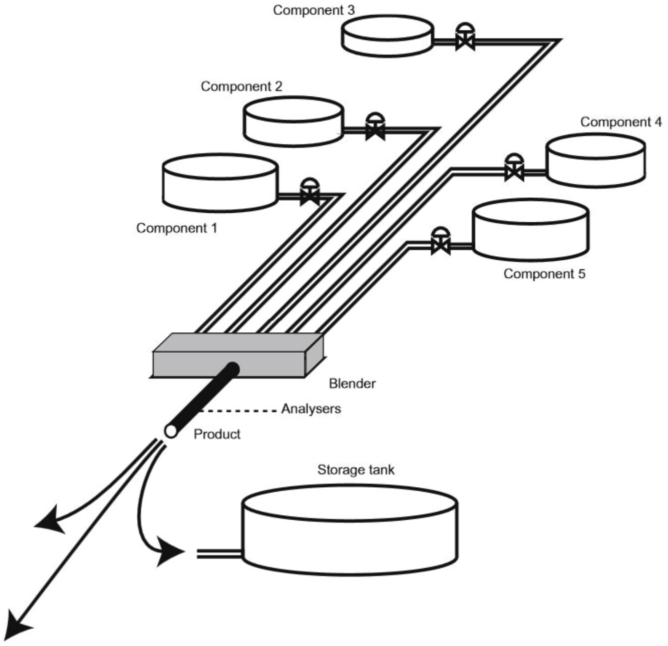

Fig. 1. Blending in a refinery

Algorithm

(1) Chose any smooth increasing functions y_1, y_2, y_3, defined on some interval $[0, T]$, with zero initial values, and final values consistent with production objectives
(2) Define

$$\eta_2^{-1} = (y_1 + y_2 + y_3)^{-1} \circ (y_1 + y_2 + y_3 + v_2)$$
$$\eta_1^{-1} = (y_1 + y_2)^{-1} \circ (y_1 + y_2 + v_1) \circ \eta_2^{-1}$$

(3) Compute the control histories according to the following formulas

$$\begin{cases} u_1(t) = \\ \quad \dfrac{\dot{y}_1 \circ \eta_1^{-1}}{(\dot{y}_1 + \dot{y}_2) \circ \eta_1^{-1}} \dfrac{(\dot{y}_1 + \dot{y}_2) \circ \eta_2^{-1}}{(\dot{y}_1 + \dot{y}_2 + \dot{y}_3) \circ \eta_2^{-1}} (\dot{y}_1 + \dot{y}_2 + \dot{y}_3)(t) \\ u_2(t) = \\ \quad \dfrac{\dot{y}_2 \circ \eta_1^{-1}}{(\dot{y}_1 + \dot{y}_2) \circ \eta_1^{-1}} \dfrac{(\dot{y}_1 + \dot{y}_2) \circ \eta_2^{-1}}{(\dot{y}_1 + \dot{y}_2 + \dot{y}_3) \circ \eta_2^{-1}} (\dot{y}_1 + \dot{y}_2 + \dot{y}_3)(t) \\ u_3(t) = \\ \quad (\dot{y}_1 + \dot{y}_2 + \dot{y}_3)(t) - u_1(t) - u_2(t) \end{cases}$$

In the second problem, accurate computation of each of the delays allows to appropriately compare measurements to predictions. This is instrumental in determining (both qualitatively and quantitatively) the true nature of each of the semi-finished products. This property can be used to define a model-based adaptive control scheme, aiming at compensating any such uncertainty (see Chèbre et al. (2010, 2011)).

Interestingly, closed-loop control of such a system is not a simple task. To illustrate this fact, one shall recall the

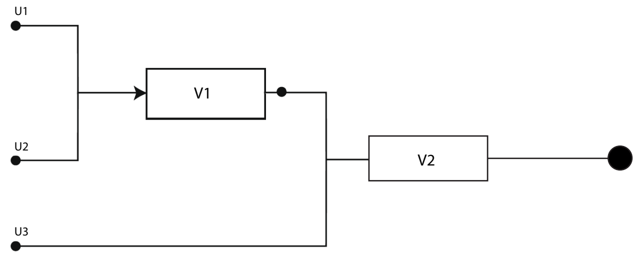

Fig. 2. A preblend network

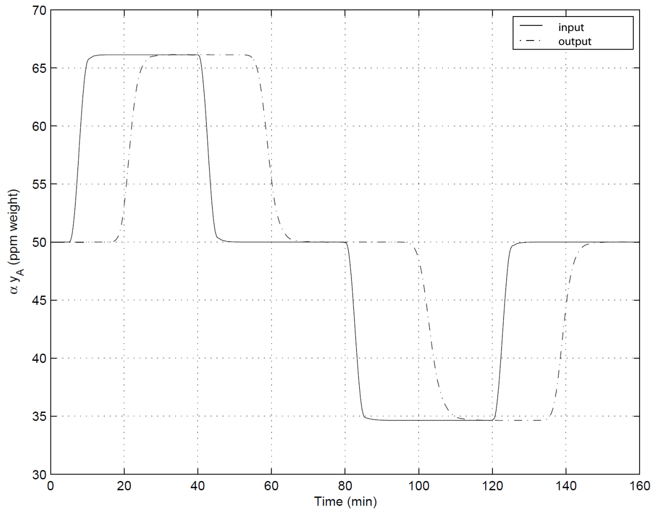

Fig. 3. Non-symmetric response for step-ups and step-downs

non-symmetric behavior being observed during step-ups and step-down responses. No such thing is observed on linear system, even with constant delays. However, one can still apply the predictor approach to control one such system. Indeed, the predictability of the delay allows one to compensate for it with good accuracy. The generalized predictor (2) is as follows

$$U(t) = K\left[e^{AD(t)}X(t) + \int_{t-D(t)}^{t} e^{A(t-s)}BU(s)ds\right] \quad (7)$$

To establish convergence of the closed-loop system, one has to introduce a Lyapounov-Krasovskii functional of the form

$$V(t) = \|X(t)\|^2 + \int_{t-D(t)}^{t} U(\tau)^2 d\tau + D(t)^2 \int_{t-D(t)}^{t} \dot{U}(\tau)^2 d\tau$$

The implicit relationship (6)-(7) between the input and the delay is quite involved. The delay impacts the input which impacts the delay, in a never-ending loop. This fact can be written down in the following delay functional differential equation

$$\epsilon^{(n)} + a_{n-1}\epsilon^{(n-1)} + ... + a_0\epsilon = E(\dot{D}, ..., D^{(n)}, \epsilon_t, ..., \epsilon_t^{(n-1)})$$

where ϵ is the input error, E is a polynomial at least quadratic in all its arguments, ϵ_t denotes past values of ϵ over the interval $[t-D_0, t]$ where D_0 is some fixed (positive) value, and the (a_i) coefficients can each be freely chosen using the (vector of) control gains K.

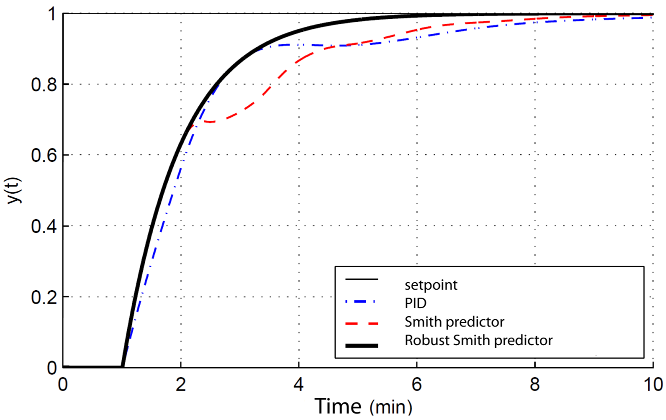

Fig. 4. Closed-loop response of a system with uncertain delay using several controllers, including a robustified Smith predictor

This equation can be analyzed, in a conservative manner, under the form of a functional inequality, using a classic analysis result, the Halanay inequality (Halanay (1966); Liu et al. (2011)). In its common form, the Halanay inequality addresses the following problem

$$\dot{x}(t) \leq -ax(t) + b \max x_t, \quad t \geq 0$$

By extending it to equations of the form

$$\dot{x}(t) \leq -ax(t) + b \max x_t + c\exp(-\gamma t), \quad t \geq 0$$

in higher dimensions (Bresch-Pietri et al. (2014)) one can formulate a small-gain theorem yielding a desirable conclusion: if the closed-loop gain K appearing in the prediction control equation (7) is sufficiently small, then the system is asymptotically stable.

4. UNCERTAIN DELAYS IN GEOGRAPHICALLY DISTRIBUTED IT SYSTEMS

We now present an example where the delay is uncertain. Industrial information technology (IT) have steadily grown and become ubiquitous over the last decades. Benefiting from this trend, data-based monitoring of processes has become considerably easier. Indeed, IT has enabled the availability of massive streams of data enabling sophisticated data analysis. However, the true situation is not that straightforward. There exists a serious and intrinsic limitation of IT that has been, so far, underestimated: erroneous dating of data. The errors stem from mis-synchronisation of the various components of the IT system.

Numerous solutions are usually implemented to mitigate the sources of mis-synchronisation. The most widely used solution is synchronisation of clocks across the IT network which is supposed to grant a unique time-reference shared by the various subsystems. Unfortunately, these synchronization procedures are built on assumptions that are impossible to guarantee, strictly speaking, and synchronisation can not be achieved with an arbitrary accuracy (Noble (2012)) [5].

[5] For example, in the state-of-the-art NTP synchronization algorithm, synchronization is only correct when both the incoming and outgoing routes between the client (the computer to be synchronized) and the server (considered as reference) have symmetrical nominal delay. Otherwise, the synchronization has a systematic bias of $\Delta/2$

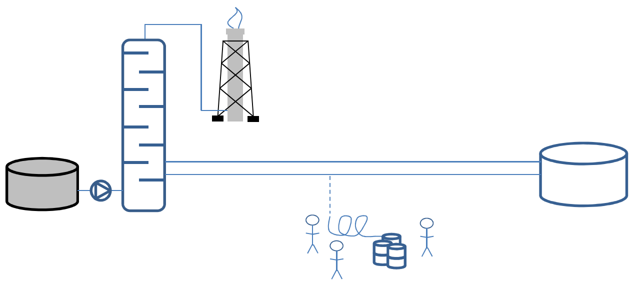

Fig. 5. Detection of leaks on a pipeline

In the general context of industrial process control (Luyben et al. (1998)), the devices needing synchronisation are in a very large number: e.g. for a refinery or a production field, at least hundreds of computers and tens of thousands of sensors are under consideration. The nature and the quality of the network employed has also a great importance. For example, in the quickly evolving "digital oilfield" applications (see e.g. Perrons (2010)), networks are composed of various types of connections (Ethernet, Wireless, Fiber, VSTA) with great variability in their bandwidth and latency.

For these reasons, relatively large mis-synchronisation of data is very common. A natural question is to determine whether mis-synchronisation is large enough to cause any problem. More precisely and quantitatively, one can formulate the following: what is the cost of working with imprecisely dated measurements?

To try to answer this question, we consider two simple applications, putting into play basic monitoring applications. We refer the interested reader to Petit (2015); Magnis and Petit (2013) for more details.

4.1 Bulk flow monitoring

Consider a pipeline that is monitored to detect leaks ((Dudek (2005); API (1995)). The situation is pictured in Figure 5. To achieve the monitoring (see Figure 6), the inlet an outlet of the transport pipe are equipped with flow meters. These devices sample the flow variable and communicate the data to a centralized system (SCADA) which applies a time stamping at the reception. Due to communication delays and their variability, the data that is available at the SCADA level is not consistent, because the flow measurements are not synchronous, although they may appear as such in the historian database, if they are received simultaneously. One could imagine that the solution would be to timestamp the data at emission (i.e. at the DAD level). However, there is no guarantee that the internal clocks of the DAD are synchronized. In practice, clocks are not synchronised [6] .

Model and state-of-the art monitoring system The usual technique employed for detecting the leaks consists in formulating a mass balance equation (Begovich et al. (2007); Fraden (2010); Geiger (2006)). Note q the inlet flow-rate, λq the outlet flow-rate, with $\lambda \in (0, 1]$ which is unknown. The loss factor is $1 - \lambda$ ($\lambda = 1$ corresponds to a no loss situation). The two DADs produce samples of the flow-rates (we note Δt the sample time), which are corrupted with noises

$$y_I[i] = q(i\Delta t) + n_i, \qquad y_O[i] = \lambda q(i\Delta t) + n'_i$$

With this modelling, an Imbalance estimator over a time-window $[0, T]$ is, simply,

$$\hat{b} = \Delta t \frac{\sum_i y_I[i] - y_O[i]}{\int_0^T q(t)dt}$$

Neglecting numerical integration error, we have:

$$\hat{b} = 1 - \lambda + \text{noise}$$

Then a simple detection algorithm is to define the alarm as follows

$$\hat{b} \geq b^*\text{: loss-alarm}$$
$$\hat{b} < b^*\text{: no loss-alarm}$$

The threshold b^* is the only parameter of this algorithm.

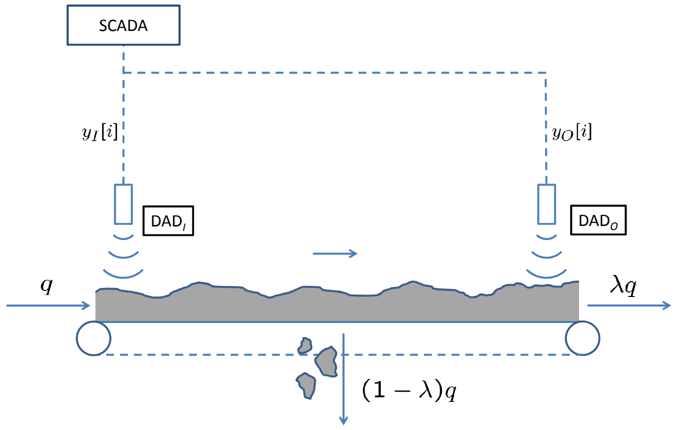

Fig. 6. Bulk-flow monitoring

A priori bound without dating uncertainty From information theory, it is possible to derive a lower bound on the variance of estimators of the parameter λ. For this, a particular stochastic setup is considered, without loss of generality.

Classically, the Cramér-Rao bound (see Frieden (2004)) expresses a lower-bound on the variance of any unbiased estimator of λ obtained from a given set of measurements X distributed according to a known probability density function.

Assuming a large number of samples are available and considering the limit case $\Delta t \to 0$, one deduces the handy formula

$$\text{Var}(\hat{\lambda}) \geq \frac{R'}{\|q\|_2^2} \tag{8}$$

where $\|q\|_2^2 = \int_0^T q(\tau)^2 d\tau$, for $[0, T]$ covering all the sample times, and R' is the power spectral density of the measurement noise corrupting the value of the data.

A priori bound with *dating uncertainty* Now, to account for dating uncertainty, we introduce noise in the sampling instants. Straightforwardly, we consider

$$y_I[i] = q(i\Delta t) + n_i, \qquad y_O[i] = \lambda q(i\Delta t + w_i) + n'_i$$

where Δ is the difference between the forward and backward travel times.

[6] We refer the interested reader to the introductory discussion on the flaws of NTP synchronisation algorithm and the weakness of the signals received from atomic clocks.

with w_i is a random value.

To quantitatively study the impact of the uncertainty, the following stochastic modeling (see Magnis and Petit (2013) for details) can be introduced. The dating uncertainties w_i are independent identically distributed centered Gaussian variables, and the overall noise is Gaussian and centered. In principles, this allows one to perform explicit computations for the probability law of accurately detecting losses and generating false alarms.

The uncertainties creates ambiguity which grows with the variance of w. Note $(\sigma_w)^2 = R_w/\Delta t$ the variance of the centered Gaussian noise w. Developing (at first order)

$$y_O[i] = \lambda q(i\Delta t) + \lambda w_i \dot{q}(i\Delta t) + n'_i$$

Then, for small values of σ_w, the dominant term in the expansion of the variance (in powers of σ_w) is, using a convenient limit $\Delta t \to 0$

$$\mathrm{Var}(\hat{\lambda}) \geq \frac{R'}{\|q\|_2^2} + R_w \lambda^2 \frac{\|q\dot{q}\|_2^2}{\|q\|_2^4} \tag{9}$$

where $\|q\|_2^2 = \int_0^T q(\tau)^2 d\tau$, $\|q\dot{q}\|_2^2 = \int_0^T q(\tau)^2 \dot{q}(\tau)^2 d\tau$, for $[0,T]$ covering all the sample times. Interestingly, (9) can be compared to (8), stressing the additional error due to dating uncertainty. If the signal q is constant, then the second factor is zero. Dating uncertainty is negligible. Otherwise, this factor can overwhelm the first (noise) factor.

Application example Under the assumption $R_w < \Delta t^3/\gamma^2$ (see Petit (2015) for details why this limits interlacing of data). We now consider a periodic signal $q(t) = 1 + \frac{1}{2}\sin(2\pi N t)$ where N is a given frequency. For any setup, there exists a critical frequency above which the error due to noise is overwhelmed by the error due to the mis-synchronisation as computed in (9). Intuitively, the higher N, the higher the sensitivity to dating uncertainty. Indeed, for a time horizon $[0,T]$, one has $\dot{q}(t) = \pi N \cos(2\pi N t)$ which shows that $\|q\dot{q}\|_2^2$ is an increasing function of N.

When the variance of w is increased, the contribution of dating uncertainty to the variance in the Cramér-Rao bound is dominant. This is clear from (9). This can be observed in Table 1. Typically, this is the case when approx. 0.2% of the dating uncertainty is larger than Δt in absolute value (here $\Delta t = 0.01$, $N = 14$). Signals are reported in Figure 7.

4.2 Allocation factor: Data Validation and Reconciliation

The second monitoring application that we wish to discuss stresses the difficulties of operating geographically distributed IT systems (see Préveral et al. (2014) for more details). A daily task that production engineers must perform in the oil industry is Data Validation and Reconciliation (DVR). After having gathered production data consisting in redundant real-time measurements of flow-rates (and possibly pressures, temperatures) from sensors placed in various locations in the production networks, the task consists in producing best estimates of the production of each well, by an analysis of the data and their comparison through mass balance equations. Determining each well production level is important for hydrocarbon accounting, to detect possible production network leaks, and very importantly, to validate geophysics studies modeling the mid-term or long-term behavior or the producing reservoir. In theory, this task boils down to a linear data analysis problem, as is shown below.

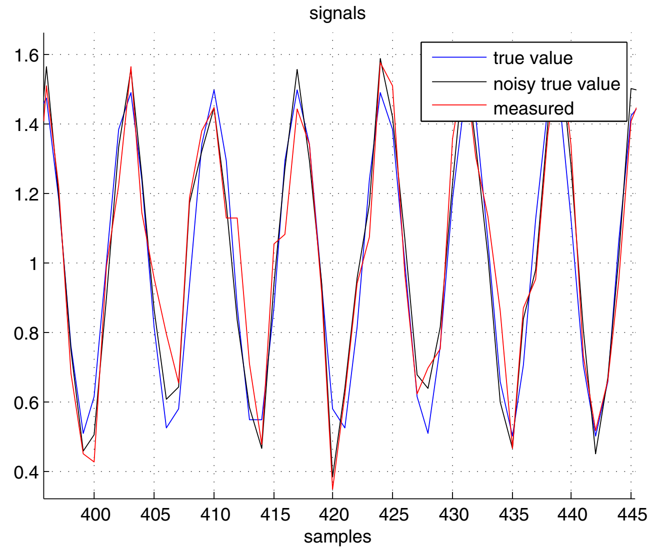

Fig. 7. Signals for which dating uncertainty is causing as much error on the loss detection as measurement noise

A case study: production network For a given production network, the DVR problem can be simply modeled as follows (in its simplest form). First, one shall partition the flow-rates as

$$x = \begin{pmatrix} x_m \\ x_u \end{pmatrix}$$

where x_m are measured flow-rates, and x_u are unmeasured flow-rates. From conservation principles (material balance), one has

$$Ax = 0 \tag{10}$$

where A is a network representation matrix (each of its elements is 0, 1 or -1).

Then, by factoring the material balance (10) as

$$A_m x_m + A_u x_u = 0$$

and by introducing some uncertainty to account for measurement noise

$$y = x_m + e$$

where e is a zero-mean (un-correlated) Gaussian noise of standard deviation σ, the DVR can be reformulated as the following constrained optimization problem

$$\begin{array}{c} \min \\ x \\ \text{s.t. } Ax = 0 \end{array} \sum_i \left(\frac{y_i - x_i}{\sigma_i}\right)^2 \tag{11}$$

Using the QR decomposition

$$A_u = QR = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix} \begin{pmatrix} R \\ 0 \end{pmatrix} \Pi$$

and noting

$$S = Q_2^T A_m, \quad \Sigma = \mathrm{diag}(\sigma)$$

in the so-called *observable case* (Narasimhan and Jordache (1999)), the solution to the DVR (11) are the estimates

% outside $[-\Delta t, \Delta t]$	error due to noise	error due to dating	Cramér-Rao bound
31.7	9.4e-06	86.1e-06	95.5e-06
13.3	9.4e-06	38.3e-06	47.7e-06
4.5	9.4e-06	21.5e-06	30.9e-06
1.2	9.4e-06	14.5e-06	23.2e-06
0.3	9.4e-06	9.5e-06	18.9e-06
0.04	9.4e-06	7.0e-06	16.4e-06
0.0	9.4e-06	5.4e-06	14.8e-06

Table 1. Error Variance due to dating uncertainty can overwhelm noise

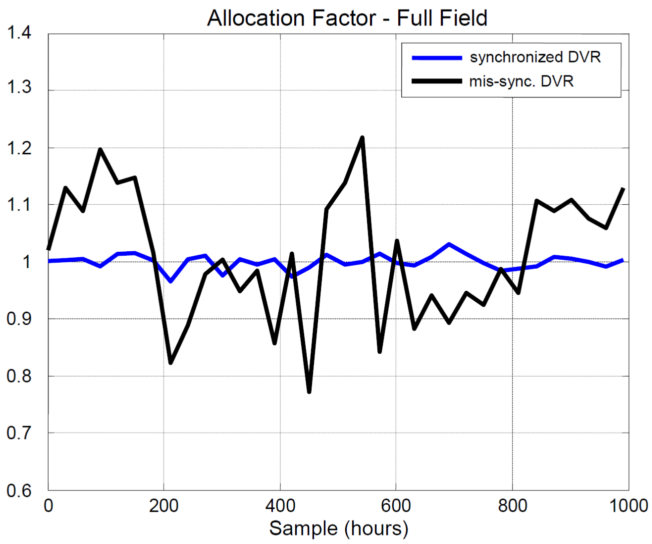

Fig. 9. Allocation factor

$$\hat{x}_m = y - \Sigma S^T (S \Sigma S^T)^{-1} S y$$
$$\hat{x}_u = -\Pi^{-1} R^{-1} Q_1^T A_m \hat{x}_m$$

which are unbiased normally distributed estimates. In details, $\hat{x}_m$ is called the reconciliation of measured variables, and $\hat{x}_u$ is the coaptation for unmeasured variables.

Considering the network pictured in Figure 8, simulation data have been treated in a DVR scenario. The sensors have various level of uncertainty. For example Well 5 has a Multiphasic Flow Meter while two export stations are measured accurately. Other sensors are deployed, providing mid-to-large uncertainty measurements.

Several mis-synchronisation have been introduced in the data. Typical synchronisation errors due to heterogeneous databases are introduced. Some flow-rates are downsampled and averaged to daily values, which also introduces some lag. Delays are present on flow-rate measurements.

Averaged values of individual well flow-rates are consistent with true means, but produce false warnings. On the other hand, the key production indicator which is the allocation factor (ratio of total well flow estimate to total measured bulk/fiscal flow) is wrongly estimated (see Figure 9). Instead of the true value which kindly oscillates in the vicinity of 1, spurious oscillations appear in the reconstructed value. Reality is much smoother than DVR estimates suggest. The spurious oscillations are due to mis-synchronisation of data produced by the IT system.

5. NON-CAUSALITY OF COMMUNICATION: REPEATER SYSTEM

We end this panorama with a system subjected to a non-causal delay. To communicate data in very adverse environments, (such as inside an oil well), repeaters can be used to overcome obstructing medium features or to increase the range. Often, data is used to monitor or to control downhole production or operations (drilling), which reveals quite a challenge in view of available communication capabilities. Among the limiting factor are bandwith and delay.

To understand better the issues raised by this kind of communication technologies, let us consider a drilling setup, under a simplified form. Downhole measurement are to be transmitted to the surface to determine relevant control actions. Cutting is realized at the bottom of the well thanks to the rotation of a cutting device and weight on bit. Both variables are controlled from the surface, using an electromechanical transmission and a counterweight. The rotation speed and the weight on bit are governed by complex dynamical equations. Assume that these are perfectly known. Then, a closed-loop control is necessary to achieve good tracking of transient and disturbance rejection Aarsnes et al. (2014); Di Meglio et al. (2014).

Bottom-hole information are transmitted to the surface using a collection of repeaters, see Figure 10. The medium through which waves propagate is the metal of the rods. Damping results from interaction with surrounding medium (mud, oil, rocks, cuttings). The repeaters are used to cancel the negative effects of damping of the transmitted signal. In details, the very narrow bandwidth and fault of transmission imply that information is not transmitted to the surface in a straightforward manner, from repeater 1 to repeater 2, and so on. Rather each packet of data follows a complex path, consisting of leapfrogs (from repeater 4 to repeater 6 for example). As a result, some data will be transmitted to the surface faster than others. Some will eventually take a long time to be transmitted. To illustrate this, we report simulation of delay variations in Figure 11. As appears, the condition $\dot{D} < 1$ is violated on several occasions. The interested reader can refer to Depouhon and Detournay (2014); Di Meglio and Aarsnes (2015) for a comprehensive model of the drilling/cutting process at stake.

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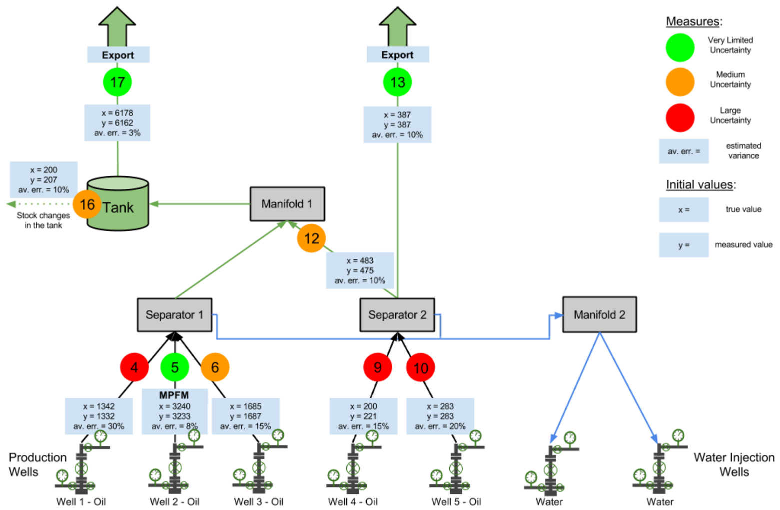

Fig. 8. A production network

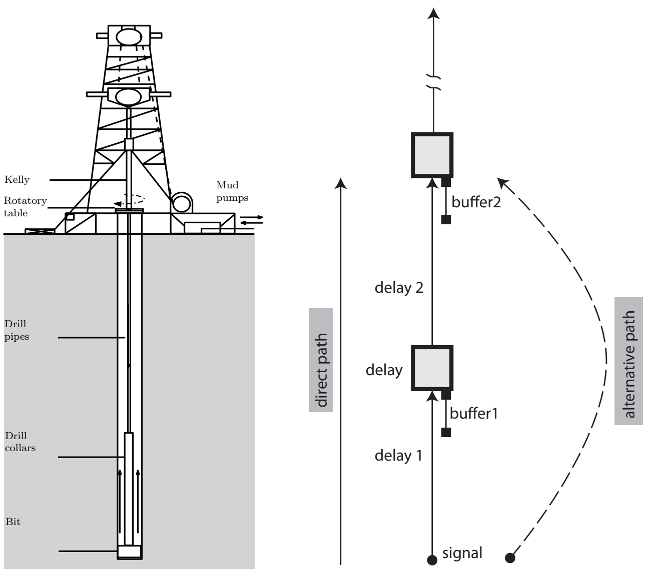

Fig. 10. Repeaters system for drilling operations

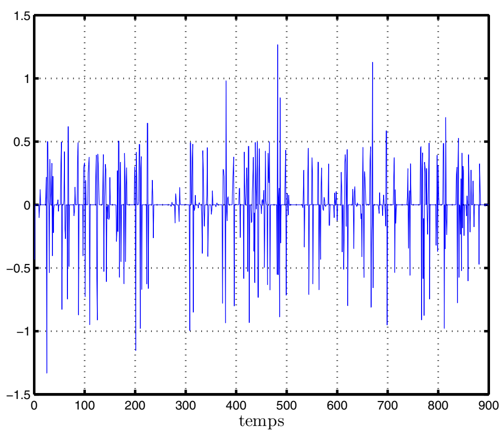

Fig. 11. Variations $\dot{D}$ of the delay caused by the repeaters of the communication system

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