PI CONTROLLERS PERFORMANCES FOR A PROCESS MODEL WITH VARYING DELAY

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Abstract

Varying delay systems represent a serious challenge in many facets of process control. A frequent issue that arises in practice is introduced by transportation delays in fixed lengths pipes at speed which varies with setpoints. Many classic control techniques can be used to deal with constant delays systems but they do not specifically address this structural delay variability. In this paper we present a process model (Diesel Hydrodesulfurization) that features this delay variability and explore robustness properties of a wide panel of PI controllers. A conclusion is that the recent method proposed by Tavakoli and Fleming compares favorably with all others, including Smith predictors, when the delay variation is not known.

1 Introduction

In spite of all of the advances in process control over the 50 last years, the PI controller is still the most commonly encountered controller in the process industry. Though PI controllers can address delays in the systems dynamics, one of the serious practical limitations of this SISO controller is reached when dealing with time-varying delays. This situation can be problematic when dealing with transportation delays in fixed lengths pipes at speed which varies with setpoints. Indeed, these systems are ubiquitous in refineries, blending networks, and other systems that imply not negligible transport phenomena.
In a first attempt to solve this problem we explore the robustness properties of a wide panel of PI controllers including the newly proposed controller by Tavakoli and Fleming [7].
After briefly presenting the tuning methods for the PI controllers under consideration (and their key properties), we compare the obtained performances on a simplified hydrodesulfurization process model we use as test case.

2 PI controllers tuning rules

We denote the process model and controller transfer functions:

$$G(s) = \frac{Ke^{-\delta s}}{\tau s + 1}\ , \quad G_c(s) = K_c\left(1 + \frac{1}{sT_i}\right) \tag{1}$$

Tavakoli-Fleming tuning rule (TF) In [7] the authors proposed an optimal method based on a dimensional analysis and numerical optimisation techniques, for the tuning of the PI controllers for first order plus dead time systems (FOPDT). This dimensional analysis leads to relations:

$$KK_c = \ g_1\left(\frac{\delta}{\tau}\right), \quad \frac{T_i}{\delta} = \ g_2\left(\frac{\delta}{\tau}\right) \tag{2}$$

Functions g_1 and g_2 in (2) are determined for a step change in the setpoint so that the integral of the absolute error is minimized. To ensure closed loop robustness, two constraints guarantee a minimum gain margin of 6 dB and a minimum phase margin of 60°. Then genetic algorithms are used to find the best values for each $\frac{\delta}{\tau}$. Eventually functions g_1 and g_2 are determined using curve-fitting techniques:

$$\begin{aligned} KK_c &= 0.4849\,\frac{\tau}{\delta} + 0.3047 \\ \frac{T_i}{\tau} &= 0.4262\,\frac{\delta}{\tau} + 0.9581 \end{aligned} \tag{3}$$

Frequency-response method by Ziegler and Nichols (ZN) This design is based on the knowledge of the *ultimate gain* K_u and *ultimate period* T_u, two parameters that characterize the process dynamics [9]. K_u et T_u can be determined by a relay feedback as shown in [1]. Ziegler and Nichols then studied on a simple real process with a proportional controller, both the effect of disturbance and the effect of load change. Their conclusion was that a good compromise between large offset and large amplitude decay ratio was to choose the tuning giving an amplitude decay ratio of 0.25. An experience of load change is used again to find the best

response with a PI controller where the gain controller is $K_c = 0.45K_u$. The best response was given by an integral time $T_i = T_u/1.2$.
This method gives good results when the dead-time is short. When there is a large dead-time, the closed loop keeps robust but parameters of the controllers are de-tuned, the response is then very loose.

Cohen and Coon tuning formula (CC) Cohen and Coon presented in [2] a method to determine the adjustable parameters for a desired degree of stability.
The tuning is obtained with a theoretical study of a FOPDT system with a dimensionless equation. Harmonics in response after a Heaviside step are neglected and the amplitude ratio of the fundamental is set to 0.25. The integral time is determined with the objective of a 0.25 amplitude ratio and a compromise between a minimum control area and a maximum stability.
The Cohen-Coon method has small gain margin and phase margin when the process dead-time is short. This problem decreases when the dead-time of the process increases, this is why the (CC) tuning design is often used with processes that presents a large dead-time.

Refinements of the Ziegler-Nichols tuning formula (RZN) The design was proposed by Hang, Åstrom and Ho in 1991 [3]. Their tuning formula comes from a dimensional analysis where the dimensionless variables used are the scaled process gain $\kappa = KK_u$ and the scaled dead-time $\Delta = \frac{\delta}{\tau}$. A step response with 10% overshoot and 3% undershoot is required and defines the tuning rule.

Smith predictor (Smith) In 1957, Smith presented a control scheme for single-input single-output systems, which has the potential of improving the control of loops with dead-time (see [5] for example). It is known that Smith predictor gives good results when the model is correctly identified.
The Smith predictor can be seen as four blocks: the internal controller, the process, the process model and the process model without delay. The internal controller can be a PI controller. An open loop control is first obtained, based upon an undelayed prediction, the controller being tuned from the model without delay. Feedback action is provided through the (possibly filtered) difference between the prediction (including the delay) and the real measurement, that is added to the setpoint.

3 Process model and varying delay

Diesel Hydrodesulfurization Hydrodesulfurization is a process met in all refineries for various fluids. Here, we are looking at the desulfurization of an intermediate cut that enters the composition of diesel fuels.
For a real process, the feed to be desulfurized is mixed with a gas (essentially hydrogen). This mixture is preheated against the reactor outlet, then heated in a furnace, and is processed through the reactor. Downstream, the mixture is cooled and flashed. The gas phase is treated and then partially recycled: combined with an hydrogen make-up, it constitutes the gas to be mixed with the feedstock. The liquid phase is splitted, then cooled before being sent to the diesel pool for blending.
The operating plan we are using is the following: sulfur in the desulfurized product must be controlled at 50 ppm weight. The feed flowrate (straight run diesel, about 300 ppm weight sulfur) is equal to 200 t/h. The feed flowrate and composition change. The reactor inlet temperature is used to compensate for these disturbances. From a control point of view, the output is the sulfur concentration of the desulfurized product, the input is the reactor inlet temperature.
Some simplifying assumptions are made

- The reactor inlet temperature can be given arbitrary values instantaneously. This is not a very strong assumption: for real processes, this temperature is easily and quickly controlled by a regulatory controller acting upon the fuel flowrate (fuel to be burnt in the furnace).
- Light components are instantaneously and totally removed from the liquid in the separator located downstream the reactor. No heavy component is withdrawn in the vapor.
- The splitter is seen as a simple mixing drum.
- The ratio between feed and gas (recycle+make up) is kept constant.
- The composition of the gas mixed with the liquid feed is constant. Otherwise stated, we do not consider the variations of the hydrogen fraction in the gas, that are due to the recycle.

These assumptions allow us to limit the usage of energy balances to the reactor part. They do not oversimplify the problem, so that the conclusions we give on a model are valid for a real process. Releasing the last two assumptions would not lead to qualitatively different results.
The simplified model keeps the two main characteristics we wanted to isolate for the tests: besides nonlinearities providing a variable gain, transportation through piping gives us a variable delay. Figure 1 shows the behavior of the outlet reactor and the outlet drum in open-loop when the feed flowrate varies. We denote, especially that the delay is varying from 15 min to 25 min.

Reaction We present a simplified diagram of an hydrodesulfurization unit on Figure 2.
The reaction is of the form

$$2A + B \rightarrow 2C + 2D \tag{4}$$

with $A = RSH$, $B = H_2$, $C = R$ and $D = H_2S$.

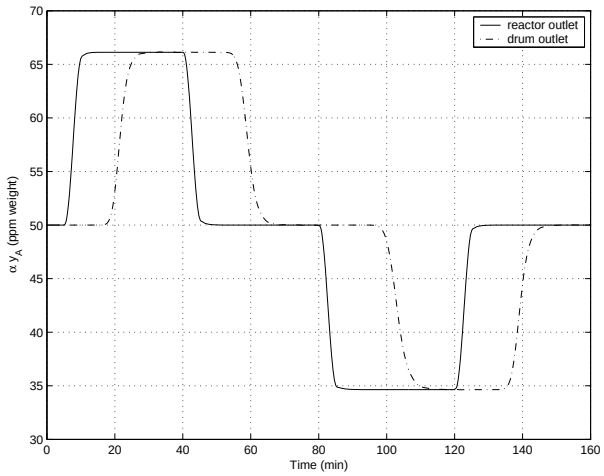

Figure 1: Weight fraction of RSH. Open-loop test.

Balance equations The model of the reactor is a plug-flow model with diffusion of energy and matter. We assume that the pressure profile inside the reactor is constant. The state of the model is only the molar fraction of the two reactants and the temperature inside the reactor. Molar fractions and energy balances are given by

$$\begin{aligned}
\frac{\partial x_A}{\partial t} &= v_{mol}\left(-\frac{F}{\Omega}\frac{\partial x_A}{\partial z} + r(.)\,(2 - x_A)\right) + D\frac{\partial^2 x_A}{\partial z^2} \\
\frac{\partial x_B}{\partial t} &= v_{mol}\left(-\frac{F}{\Omega}\frac{\partial x_B}{\partial z} + r(.)\,(1 - x_B)\right) + D\frac{\partial^2 x_B}{\partial z^2} \\
\tau_T\frac{\partial T}{\partial t} &= -\frac{FC_P}{\Omega}\frac{\partial T}{\partial z} + \Delta H\, r(.) + D_T\tau_T\frac{\partial^2 T}{\partial z^2}
\end{aligned} \tag{5}$$

where $T(0,t)$ is the control and $x_A(0,t)$ and $x_B(0,t)$ are constants (that can be used as disturbances). The term τ_T stands for a ρC_p-like term taking into account the fluid, the catalyst and the metal of the reactor. We assume that the separation downstream the reactor is perfect and modelled with the algebraic equations

$$y_j = \frac{x_j}{x_A + x_C} \quad \forall\, j \in \{A, C\} \tag{6}$$

We assume further that piping between the outlet of the separator and the inlet of the drum generates a 15 minutes delay when the feed flowrate is constant at the reference value and the mass fraction of A is stabilized at 50 ppm weight. The model is a transport equation:

$$\frac{\partial y_A}{\partial t} = -\frac{F^{drum} v^t_{mol}}{\Omega^P}\frac{\partial y_A}{\partial z} \text{ (from separator to drum)} \tag{7}$$

As there is no reaction in the drum, the model we propose is a simple mixer:

$$\frac{dy_A}{dt} = -\frac{F^{drum}}{N^{drum}}\left(y_A^{in} - y_A\right) \text{ (in the drum)} \tag{8}$$

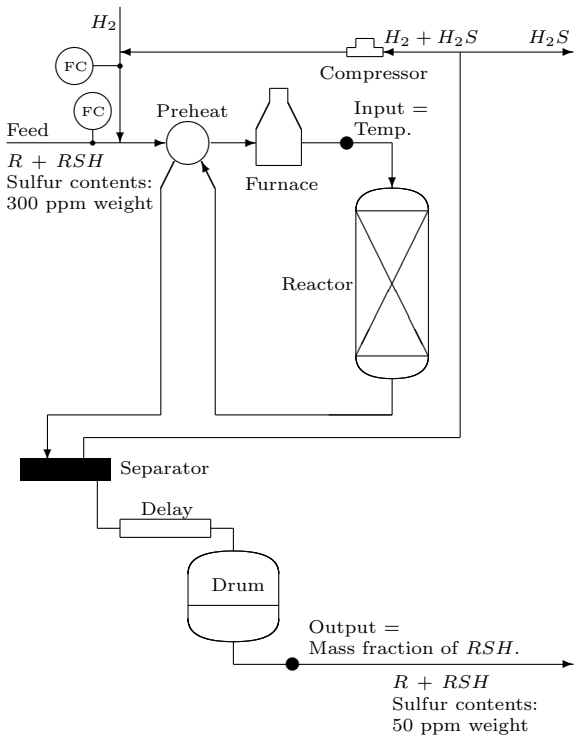

Figure 2: Simplified diagram of an hydrodesulfurization unit.

The kinetics of the reaction is denoted

$$r(.) = r\,(T, x_A, x_B) = k\exp\left(-\frac{E_a}{RT}\right) x_A\, x_B \tag{9}$$

To simulate this process model, we use a classical 1D-discretization scheme for equations (5) and (7) with 30 elements for the reactor.

4 Control model and simulations

We choose to identify the process as a first order plus dead-time system using **ISIAC**, the identification software of Institut Français du Pétrole [8]. The control model thus obtained is valid around the operating point:

$$\begin{aligned}
y_r &= \alpha\, 50 \text{ ppm molar} \\
u_r &= 623 \text{ K}
\end{aligned}$$

where α is a constant used to convert weight fractions to molar fractions. The linear input-output model is noted:

$$\dot{y}(t) = -\frac{1}{\tau}(y(t) - y_r) + \frac{K}{\tau}(u(t-\delta) - u_r) \tag{10}$$

where **ISIAC** identification gives:

$$\begin{aligned}
K &= -2.17 \text{ ppm.K}^{-1} \\
\tau &= 2.5 \text{ min} \\
\delta &= 15.7 \text{ min}
\end{aligned}$$

Symb.	Quantity	Unit
D	Diffusion coef. for matter	$m^2.s^{-1}$
D_T	Diffusion coef. for temp.	$m^2.s^{-1}$
E_a	Activation energy	$J.mol^{-1}$
F	Molar flow at z	$mol.min^{-1}$
F^{drum}	Molar flow inside the drum	$mol.min^{-1}$
k	Rate constant	$mol.m^3.s^{-1}$
K	Static gain	$ppm.K^{-1}$
K_c	Controller gain	$K.ppm^{-1}$
R	Gas constant	$J.K^{-1}.mol^{-1}$
t	Time	min
T	Temperature (Temp.)	K
T_i	Integral time	min
v_{mol}	Molar volume in the reactor	$m^3.mol^{-1}$
v^t_{mol}	Molar volume in the pipe	$m^3.mol^{-1}$
x_A	Molar fraction of A	
x_B	Molar fraction of B	
y_A	Molar fraction of A after sep.	
y_B	Molar fraction of B after sep.	
z	Length unit	m
α	ppm weight $\rightarrow$ molar	
δ	Delay	min
ΔH	Reaction enthalpy	$J.mol^{-1}$
Ω	Reactor's section	m^2
Ω^P	Pipe section	m^{-1}
τ	Time constant	min
τ_T	Pseudo time constant	$J.K^{-1}.m^{-3}$

Table 1: Nomenclature.

The limit gain and limit period are obtained with relay controller on the process:

$$\begin{aligned} K_u &= -0.594 \text{ K.ppm}^{-1} \\ T_u &= 38 \text{ min} \end{aligned}$$

Robustness with delay changes The varying delay is due to the varying feed flowrate, this leads us to test robustness by introducing changes in the flowrate F at the inlet of the reactor. F_{ref} is the feed flowrate which has been used for the model identification. The simulation involves five steps:
Step 1: when $t \in [0, 5]$, $F = F_{ref}$. **Step** 2: when $t \in [10, 145]$, $F = 1.2F_{ref}$. **Step** 3: when $t \in [150, 295]$, $F = F_{ref}$. **Step** 4: when $t \in [300, 445]$, $F = 0.8F_{ref}$. **Step** 5: when $t \in [450, 600]$, $F = F_{ref}$.
Figure 3 shows the sulfur mass fraction at the drum outlet. During **Step 2**, as the flowrate is more important, the dead-time decreases. All the controllers make the output converge towards the reference. ZN tunings gives the worst result. Good responses can be achieved by three different PI controllers. The two first ones, respectively TF and RZN lead to similar responses while the CC method, although different, converges as fast as the later ones. The Smith predictor response is faster than the PI responses. Dead-time identification errors create small oscillations on the output. The magnitude of oscillations increases with delay identification error, if this error becomes too large, the Smith predictor destabilizes the output. When the delay identification error is known to be large, Smith can be used with de-tuned controllers and with an important filter time constant. The response thus obtained is worse than the response given by the best PI controller.
Step 3 emphasizes the superiority of the Smith predictor when the delay is accurately identified. Indeed, the Smith predictor brings the output at setpoint very quickly. After it, the three best PI controllers are the same than those in step 2.
During **Step 4**, as the flowrate is less important, the dead-time increases. All the controllers make the output converge towards the reference and the three best PI controllers are the same than those in step 2. The Smith predictor response keeps stable but the response oscillates around the setpoint.
During **Step 5**, the feed flowrate is equal to the reference feed flowrate, conclusions are the same than in the step 3. The Smith predictor gives better behavior than PI controllers. The three best PI controllers are the same than those in step 2.

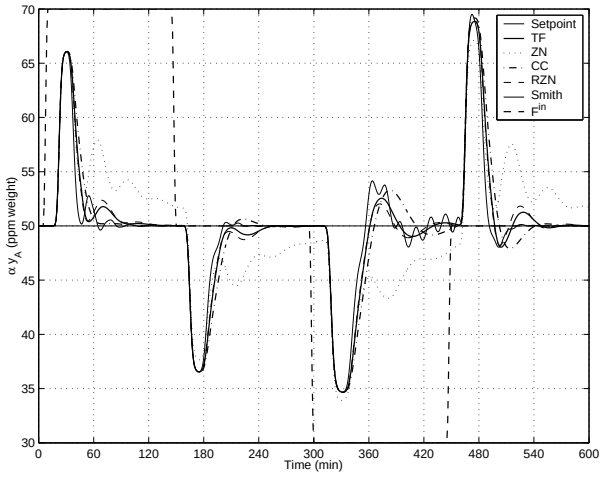

Figure 3: Weight fraction of RSH at the outlet of the drum.

Tracking We propose in this section a tracking example without varying delay where the reference changes four times within ten hours. The simulation results are presented on figure 4. At the beginning, the process is initialized on an equilibrium point with a setpoint $y_r = \alpha 50$ ppm molar. The simulation involves five steps:
Step 1: when $t \in [0, 5]$, $y_r = \alpha 50$ ppm molar. **Step** 2: when $t \in [10, 145]$, $y_r = \alpha 60$ ppm molar. **Step** 3: when $t \in [150, 295]$, $y_r = \alpha 50$ ppm molar. **Step** 4: when $t \in [300, 445]$, $y_r = \alpha 40$ ppm molar. **Step** 5: when $t \in [450, 600]$, $y_r = \alpha 50$ ppm molar.

The Smith predictor response is faster than the others for the fourth steps, and in spite of the small first overshoot, its rise time and its settling time are the shortest. The RZN and TF responses have a similar behavior. The rise time and the settling time keep fast, although slower than the Smith response. The CC response has the lowest rise time, and the response follows a sizeable single overshoot to converge towards the setpoint in the same settling time than RZN and TF.
Results are very similar to the disturbances rejection case. Again the TF tuned PI controller behaves well when compared to others. Only the Smith predictor can perform about the same.

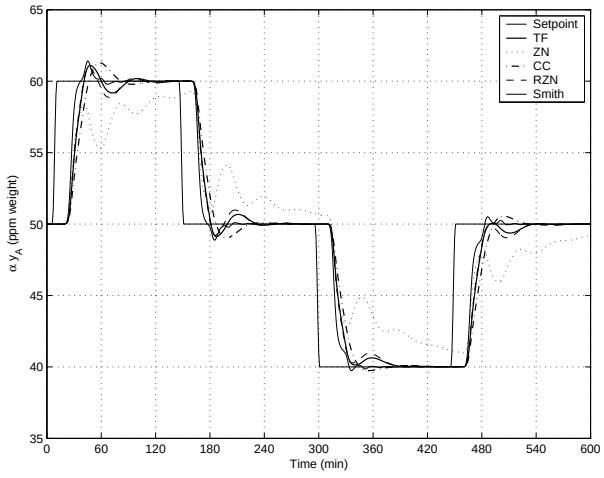

Figure 4: Weight fraction of *RSH* at the outlet of the drum. Tracking test.

5 Conclusion

The obtained results illustrate the behaviors of the process model with some different PI controllers and with the Smith predictor.
If the dead-time is accurately identified, Smith predictor can be tuned so that the obtained closed-loop response is fast. Indeed, the Smith predictor can give faster convergence than the best of PI controllers. Nevertheless, with this type of extreme tuning, a small dead-time mismatch can make the output diverge. Usually, the Smith predictor tunings will be loosen in order to avoid any divergence, at the expense of suboptimality when the delay does not vary.
When the delay is not well known, the most interesting response is obtained with the Tavakoli and Fleming (TF) PI tunings. In this situation, Smith predictor tunings must be detuned significantly, which leads to a response less effective than the TF PI tunings one.
As a conclusion, the TF PI tuning rules seem to be a good choice because of its higher stability compared to the Smith predictor in case of dead-time disturbance. Although easily implemented and effective, this controller is however not optimal when the model is accurate. As noticed before, the Smith predictor is sensitive to dead-time mismatch, and if the dead-time is varying significantly with time, the dynamic performance of the Smith predictor can be damaged. However, if an on-line dead-time estimation is applied, the Smith predictor could then be used easily with large improvement. Our current work focuses on such adaptive Smith-like predictors. Other PID tuning rules such as Lee et al. [4] and Skogestad [6] may be conceivable as well but a fair comparison would require that the D term is also considered.

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